Math Ma **Linear Functions and Local Linearity** Fall 2016

1. The table below shows information on sunrise and sunset time for Cambridge, MA in 2016:

	17 Sep	18 Sep	19 Sep	20 Sep
Sunrise	6:38 am	6:39 am	6:41 am	6:43 am
Sunset	7:09 pm	7:07 pm	7:04 pm	7:02 pm

   (a) Can you estimate when the sun will rise on September 30th?

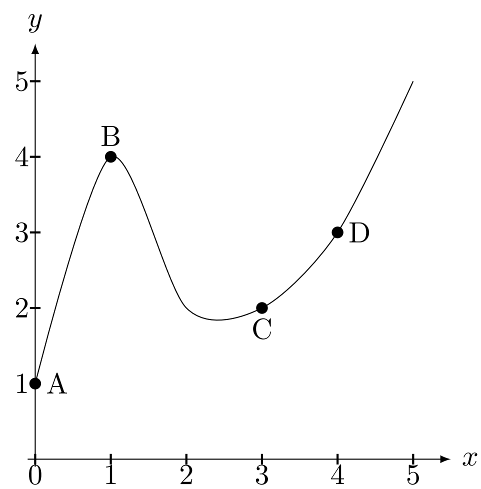
   (b) Can you estimate when the sun will rise on November 30th?

   (c) Sketch a continuous function modeling the sunrise time through the whole year.

   (d) How would you interpret your estimate in terms of graphs?

   (e) What is the meaning of the slope of secant lines in this setting?

2. Below is the graph of a function $f(x)$.



(a) Find the equation of the secant line through point A and C.

(b) Find the equation of the line through point B which is perpendicular to the line through point A and C.

(c) Find the equation of the secant line through point C and D. Can you use the secant line to approximate $f(3.2)$? How about $f(1.2)$?

3. Let $f(x) = x^2$.

   (a) Find the secant line of f through the point whose x-coordinate is 1 and the point whose x-coordinate is 2.

   (b) Find the slope of the secant line of f through the point whose x-coordinate is 1 and the point whose x-coordinate is a.

   (c) Find the slope of the secant line of f through the point whose x-coordinate is a and the point whose x-coordinate is $a + h$.

   Now let $g(x) = |x|$.

   (d) Find the slope of the secant line of g through the point whose x-coordinate is 0 and the point whose x-coordinate is a.

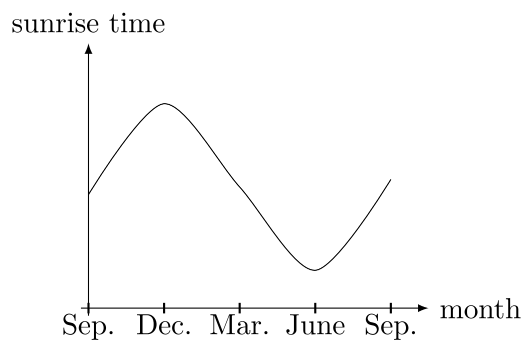
   If you have more time, try to do problem (c) with other functions you know; for example, $f(x) = x^3, \frac{1}{x}, \sqrt{x}$ or any polynomials and rational functions.

Linear Functions and Local Linearity – Solutions

1. (a) The sun will approximately rise at 7:03am.

   (b) We cannot use the information to approximate too far ahead. It would not be too inaccurate.

   (c)



   (d) We were drawing secant lines on the graph, and use the secant line to approximate values of the function.

   (e) The slope of a secant line represents the average change of the sunrise time after one day.

2. (a) We can use the two-point formula to find the equation of the secant line:

$$\frac{y-1}{x-0} = \frac{2-1}{3-0}$$

   Hence the equation of the secant line is $x = 3(y-1)$, or equivalently $y = \frac{1}{3}x + 1$.

   (b) The slopes of perpendicular lines multiply to -1, so the slope of our desired line is -3. Using point slope formula we know the equation of the desired line is

$$y - 4 = -3(x-1)$$

   or $y = -3x + 7$.

   (c) Again we may use two-point formula to find the equation of the secant line:

$$\frac{y-2}{x-3} = \frac{3-2}{4-3}$$

   Hence the equation of the secant line is $x - y - 1 = 0$. We can use the secant line $y = x - 1$ to approximate $f(3.2)$ as $3.2 - 1 = 2.2$. For $f(1.2)$, it seems not to be a good idea to approximate using this secant line.

3. (a) We want to find the line through $(1, 1)$ and $(2, 4)$. Using two-point formula, we have

$$\frac{y-1}{x-1} = \frac{4-1}{2-1}$$

   Namely, $y = 3x - 2$.

(b) We want to find the slope of the line through $(1, 1)$ and (a, a^2), which is

$$\frac{a^2 - 1}{a - 1} = a + 1$$

(c) We want to find the slope of the line through (a, a^2) and $(a + h, (a + h)^2)$, which is

$$\frac{(a + h)^2 - a^2}{(a + h) - a} = \frac{a^2 + 2ah + h^2 - a^2}{h} = 2a + h$$

(d) We want to find the slope of the line through $(0, 0)$ and $(a, |a|)$, which is

$$\frac{|a|}{a}$$

This slope is 1 if $a > 0$ and -1 if $a < 0$.