Math Ma

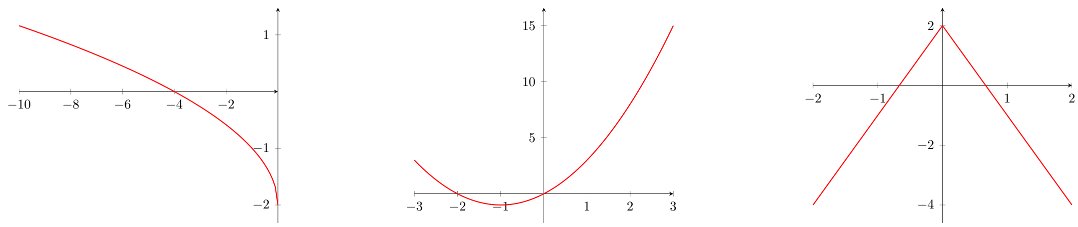
Altering Functions

Fall 2016

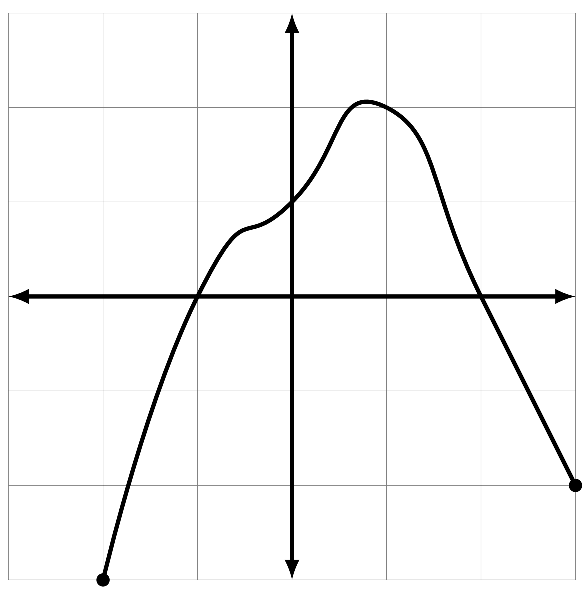
1. How would you draw the graph of the following functions?

   (a) $y = \sqrt{-x} - 2$ (b) $y = 2(x+1)^2 - 1$ (c) $y = -|3x| + 2$

Solution.



2. Here is the graph of a function f.



Fill in the following table:

The graph of a function is given. It is not easy to tell what the formula for this function is. We can use the graph to determine the value of the function at different points.

$f(-2) =$ **-3**

$f(-1) =$ **2**

$f(0) =$ **0**

$f(1) =$ **0**

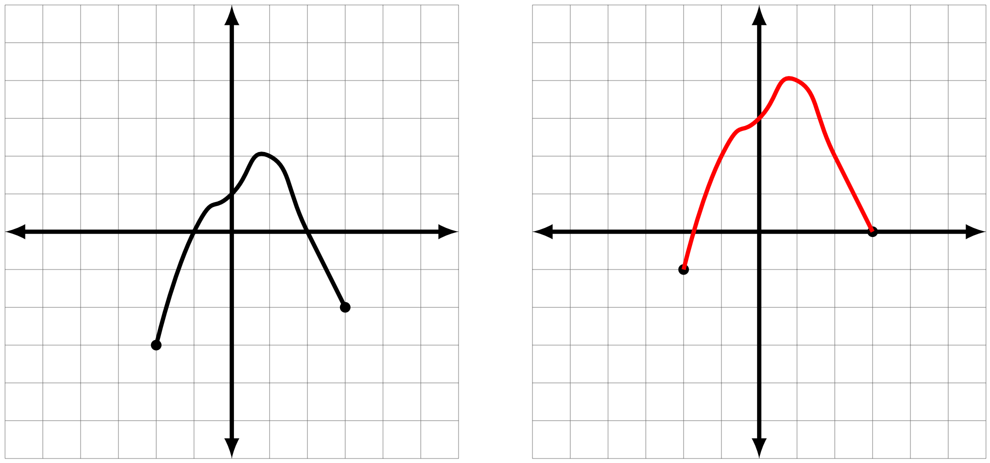
$f(2) =$ **1**

$f(3) =$ **-2**

What is the domain of f? $[-2, 3]$ ________

What is the range of f? $[-3, 2.1]$ ________

3. Define a new function $j(x) = f(x) + 2$. Sketch the graph of $j(x)$ below:



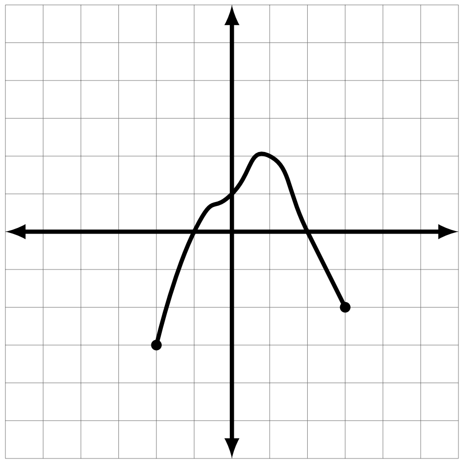
(a) How are the graphs of j and f related?

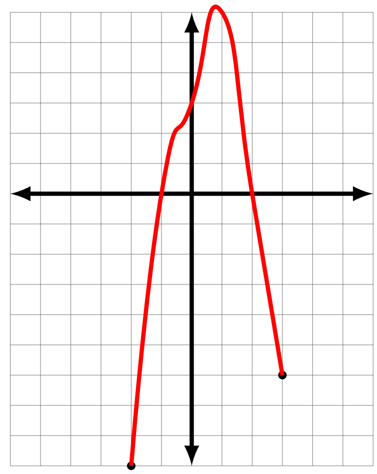
Solution. The graph of j is just a copy of the graph of f translated up 2 units. You can see a correspondence between the geometry and the algebra. The algebraic formula for J is to add 2 to the output from j. This results in shifting the entire graph up 2

(b) What are the domain and range of $j(x) = f(x) + 2$? How do they relate to the domain and range of $f(x)$?

Solution. The domain for j and f is exactly the same, $[-2, 3]$. The range for j is the range for f shifted 2 units, $[-1, 4.2]$.

4. Define a new function $k(x) = 3f(x)$. Sketch the graph of $k(x)$ below:





(a) How are the graphs of $k(x)$ and $f(x)$ related?

Solution. The graph of k is a stretched graph of f. The stretching is a factor of 3. Note how the algebraic way of writing $k(x) = 3f(x)$ shows that the outputs are being multiplied by 3. This affect on the outputs is geometrically seen as a vertical stretch.

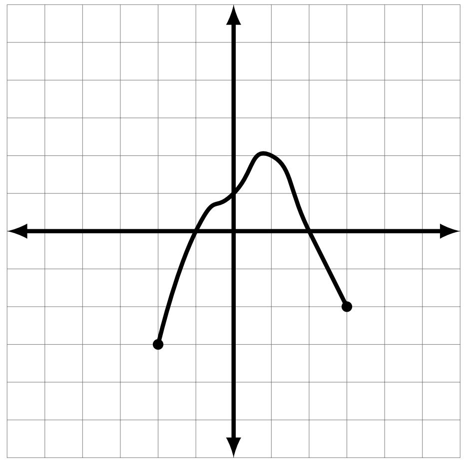
(b) What are the domain and range of $k(x) = 3f(x)$? How do they relate to the domain and range of $f(x)$?

Solution. The domain is the same. The range is $[-9, 6.3]$.

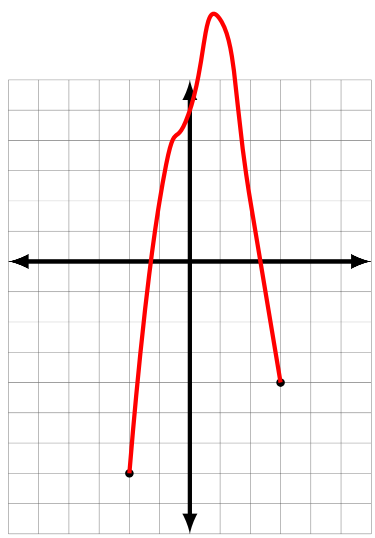
5. (a) Define a new function $G(x)$ by $G(x) = 3f(x) + 2$. Fill in the following table:

$G(-2) =$ **-7**	$G(1) =$ **8**
$G(-1) =$ **2**	$G(2) =$ **2**
$G(0) =$ **5**	$G(3) =$ **-4**

(b) *Now, use your data to sketch a graph of* $G(x) = 3f(x) + 2$.



Solution. The graph of $G(x) = 3f(x) + 2$ is shown in red below. (The graph of $f(x)$ is shown in black above.)



(c) *In words, describe how the graph of $G(x) = 3f(x)+2$ relates to the graph of $f(x)$.*

Solution. When we want to know $g(x)$ for a particular value of x, we first find $f(x)$, then multiply that by 3, and finally add by 2. This has the effect of stretching the graph of $f(x)$ vertically by a factor of 3 and *then* moving that up 2 units. The order of operations controls the order in which we do the geometric moves.

(d) *What are the domain and range of $G(x) = 3f(x) + 2$? How do they relate to the domain and range of $f(x)$?*

Solution. The domain of $G(x)$ is the same as the domain of $f(x)$, $[-2, 3]$. The range of $G(x)$ is $[-7, 8]$ (this is the range of f, stretched by a factor of 3 and then moved up 2).

(e) *More generally, how do you think the graph of $Af(x) + B$ relates to the graph of $f(x)$ if A and B are positive constants? What about when A and/or B are negative?*

Solution. In general, to get the graph of $Af(x) + B$ from the graph of $f(x)$, we first stretch the graph of $f(x)$ vertically by a factor of A, and then we move that up by B units (if B is negative, we are really moving the graph down; for instance, if $B = -5$, then we move the graph down 5 units).

If A is negative, things are a bit more complicated. For instance, if we want to graph $-3f(x)$ (so $A = -3$), we flip the graph vertically and then stretch it by a factor of 3.

6. (a) Define a new function $H(x)$ by $H(x) = f(x + 2)$. Fill in the following table:

$H(-2) = f(\underline{\mathbf{0}}) = \underline{\mathbf{1}}$ $H(1) = f(\underline{\mathbf{3}}) = \underline{\mathbf{-2}}$

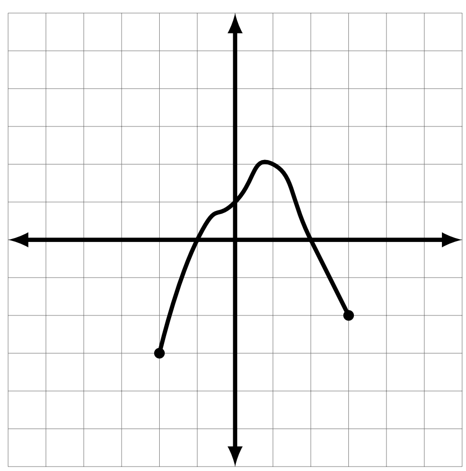
$H(-1) = f(\underline{\mathbf{1}}) = \underline{\mathbf{2}}$ $H(2) = f(\underline{\mathbf{4}}) =$ **undefined**

$H(0) = f(\underline{\mathbf{2}}) = \underline{\mathbf{0}}$ $H(3) = f(\underline{\mathbf{5}}) =$ **undefined**

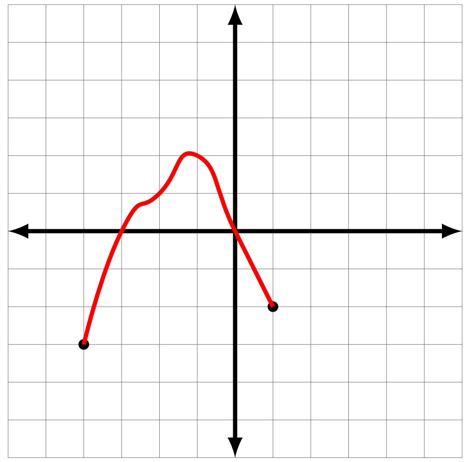
What other data might be relevant for graphing H?

$H(\underline{\mathbf{-4}}) = f(\underline{\mathbf{-2}}) = \underline{\mathbf{-3}}$ $H(\underline{\mathbf{-3}}) = f(\underline{\mathbf{-1}}) = \underline{\mathbf{0}}$

(b) *Now, use your data to sketch a graph of $H(x) = f(x + 2)$.*



Solution. The graph of $H(x) = f(x + 2)$ is shown in red below. (The graph of $f(x)$ is shown in black.)



(c) *In words, describe how the graph of $H(x) = f(x+2)$ relates to the graph of $f(x)$.*

Solution. To get the graph of $f(x+2)$, we take the graph of $f(x)$ and move it left by 2 units. The domain is being affected in this new rule. What was happening at 0 is now happening at -2; the geometric affect of this is a shift to the left by two units.

(d) *What are the domain and range of $H(x) = f(x+2)$? How do they relate to the domain and range of $f(x)$?*

Solution. The domain of $H(x) = f(x+2)$ is $[-4, 1]$. This is the domain for f moved two units to the left. The range of $H(x)$ is $[-3, 2]$, exactly the same as the range of $f(x)$.

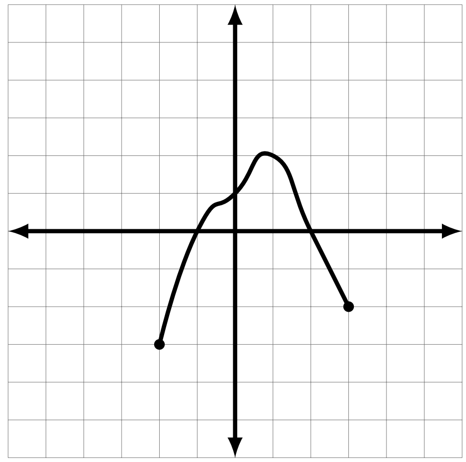
(e) *More generally, how do you think the graph of $f(x+D)$ relates to the graph of $f(x)$ if D is any constant? Do you think your rule works when D is negative?*

Solution. It is the graph of $f(x)$, translated left by D. What was happening at D is now happening at $-D$.

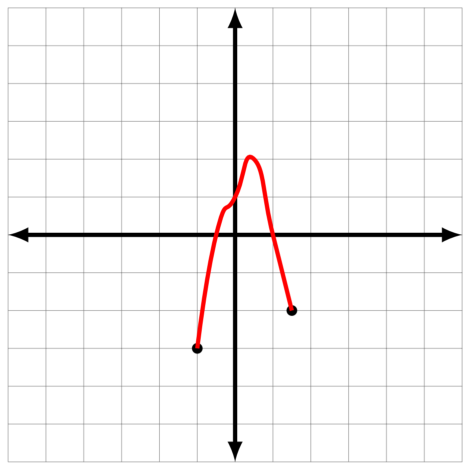
7. (a) Define a new function $F(x)$ by $F(x) = f(2x)$. Fill in the following table:

$F(-2) = f($**-4**$) =$ **undefined**	$F(1) = f($**2**$) =$ **0**
$F(-1) = f($**-2**$) =$ **-3**	$F(2) = f($**4**$) =$ **undefined**
$F(0) = f($**0**$) =$ **1**	$F(3) = f($**6**$) =$ **undefined**

(b) *Now, use your data to sketch a graph of $F(x) = f(2x)$.*



Solution. The graph of $F(x) = f(2x)$ is shown in red below. (The graph of $f(x)$ is shown in black above.)



(c) *In words, describe how the graph of $F(x) = f(2x)$ relates to the graph of $f(x)$.*

Solution. It is compressed horizontally, rescaled by a factor of $\frac{1}{2}$. That is, the graph is $\frac{1}{2}$ as wide. Notice that since we are modifying the inputs of the function it will affect the domain. What was once happening at 1 is no happening at 2.

(d) *What are the domain and range of $F(x) = f(2x)$? How do they relate to the domain and range of $f(x)$?*

Solution. The domain of $F(x) = f(2x)$ is $[-1, 3/2]$; this is the domain of f rescaled by a factor of $\frac{1}{2}$. The range of $F(x) = f(2x)$ is $[-3, 2]$, the same as the range of f.

(e) *More generally, how do you think the graph of $f(Cx)$ relates to the graph of $f(x)$ if C is any positive constant? What happens when C is negative?*

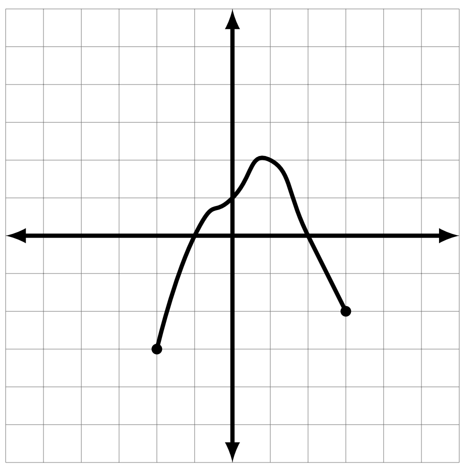
Solution. To get the graph of $f(Cx)$, start with the graph of $f(x)$. If $C > 0$, stretch the graph of $f(x)$ horizontally by a factor of $\frac{1}{C}$ (in other words, horizontal distances on the graph of $f(x)$ are $\frac{1}{C}$ times as far apart on the graph of $g(x)$).

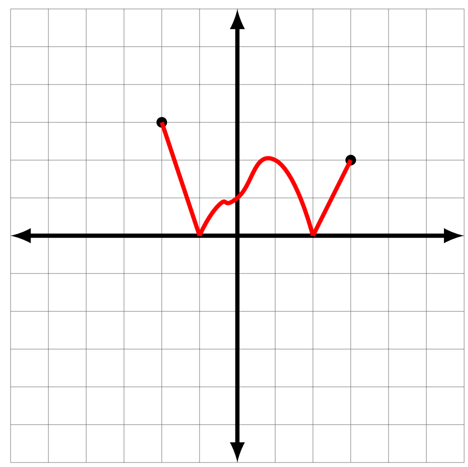
If $C < 0$, first flip the graph over the y-axis, and then stretch the graph of $f(x)$ horizontally by a factor of $\frac{1}{|C|}$.

8. (a) Define a new function $K(x)$ by $K(x) = |f(x)|$. Fill in the following table:

$K(-2) =$ **3**	$K(1) =$ **2**
$K(-1) =$ **0**	$K(2) =$ **0**
$K(0) =$ **1**	$K(3) =$ **2**

(b) Now, use your data to sketch a graph of $K(x) = |f(x)|$. Sketch the graph of $K(x)$.





(c) *In words, describe how the graph of $K(x) = |f(x)|$ relates to the graph of $f(x)$.*

Solution. The absolute value graph takes the portion of the graph that is negative and makes it positive.

(d) *What are the domain and range of $K(x) = |f(x)|$? How do they relate to the domain and range of $f(x)$?*

Solution. The domain of $K(x) = |f(x)|$ is $[-2, 3]$; this is the same as the domain of f. The range of $K(x) = |f(x)|$ is $[0, 3]$.