All About Functions!

1. This is a continuation of the Question 3 on your problem set.

   (a) Sketch a graph of the time it takes to harvest the crop of apples as a function of the number of people picking apples. Label your axes.

   (b) Is your graph continuous or discontinuous? Why did you draw your graph like that?

   (c) Which of the following glossaries can be used do describe your graph? Why did you draw your graph like that?

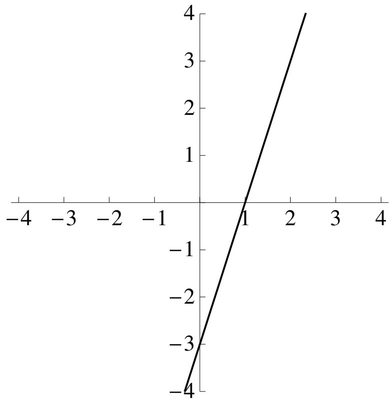
   - **positive/negative**
   - **increasing/decreasing**
   - **concave up/concave down**
   - **one-to-one**

   (d) Does your graph intersect with the horizontal axis? The vertical axis? If so, where? If not, why not?

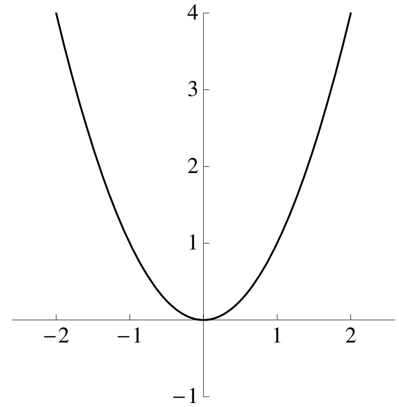
   (e) Can you draw a **continuous, positive, increasing, concave down** function?

2. For each of the following graph, think of a function $y = f(x)$ with the same graph.

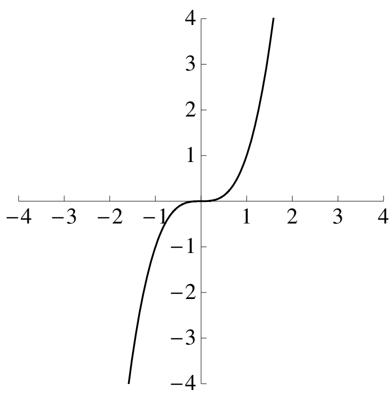
(a)



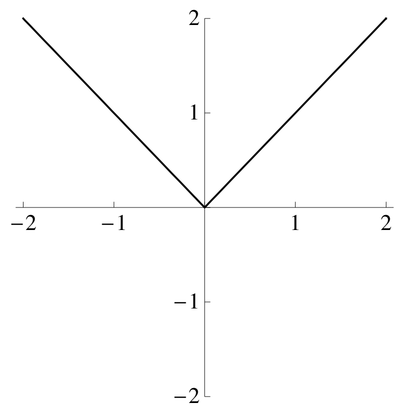
(b)



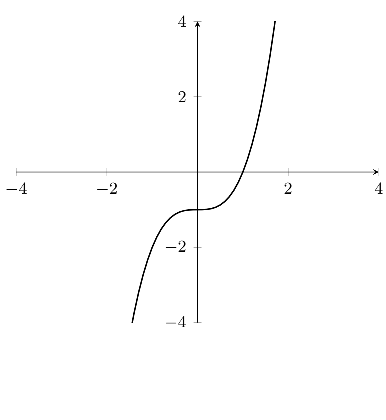
(c)



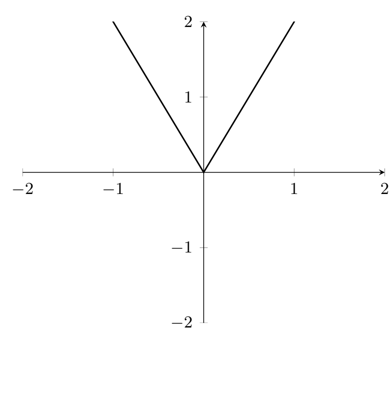
(d)



(e)



(f)



3. Sketch the graph of each of the following functions.

(a) $f(x) = -2x + 1$

(b) $f(x) = \frac{1}{x}$

(c) $f(x) = \sqrt{x}$

(d) $f(x) = \frac{1}{x^2}$

(e) $f(x) = \sqrt[3]{x}$

Try to draw (b)(d) in the same graph and compare their positions. Also try (c)(e).

Definition

- A function $f(x)$ is **even** if ____________________ or ____________________.
- A function $f(x)$ is **odd** if ____________________ or ____________________.

4. Use the definition to decide whether each of the following function is even, odd, or neither.

(a) $f(x) = -3x^4 + 6x^2 + 1$ (b) $f(x) = x^3 + x^2 + 1$ (c) $f(x) = -5x^5 - x^3$

(d) $f(x) = \frac{x^4}{x^2+1}$ (e) $f(x) = \frac{x(x^2+1)}{3x^2-2}$ (f) $f(x) = \sqrt[5]{x}$

5. Sketch the graphs of $f(x) = x$ and $g(x) = \frac{x^2}{x}$. Are the two graphs the same?

All About Functions! – Solutions

1. (a) There are many different possibilities here. You should look for some commonalities between possible solutions. Many possible solutions are both decreasing and concave down. What ever graph you draw make sure that you can justify the features of it

   (b) Both models can be reasonable. On one hand, you might say that you can never have, say $\frac{1}{3}$ of a worker; the number of workers must be a positive integer. That means that the domain of this function ought to be the positive integers, so there's no way the graph can be continuous. On the other hand, we will very often model situations like this using a continuous model. It will allow us to use the tools of calculus.

   (c) The graph should be positive, as the time always is. It should be decreasing, as more workers should save the time to harvest the crop. The graph should be a concave up graph if it is to be continuous, positive and decreasing. Finally, the graph is in principle one-to-one. However, if you think It is possible that there are more workers but time is not saved, then the graph can also be not one-to-one.

   (d) The graph cannot intersect the vertical axis: if there are no workers, then the crop will never be harvested, so 0 is not in the domain of the time function. The graph also cannot intersect the horizontal axis: no matter how many workers there are, the amount of time it takes to harvest the field will always be positive (even if it's just a split second).

   (e) No, such a function cannot exit.

2. (a) This is a line, so it should be described by an equation of the form $y = mx + b$ where m represents the slope and b represents the $y-$intercept. Therefore the equation must be $f(x) = 3x - 3$.

   (b) $f(x) = x^2$.

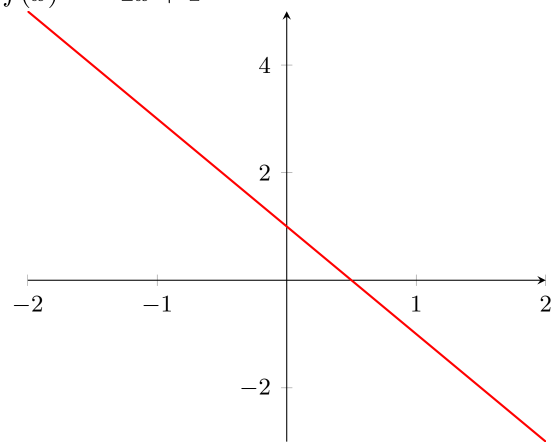
   (c) $f(x) = x^3$.

   (d) $f(x) = |x|$.

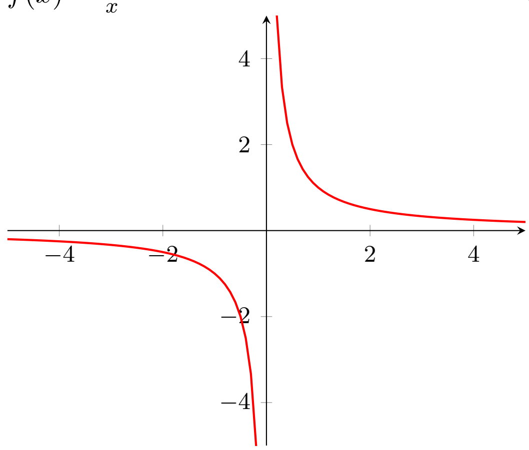
   (e) $f(x) = x^3 - 1$.

   (f) $f(x) = 2|x|$.

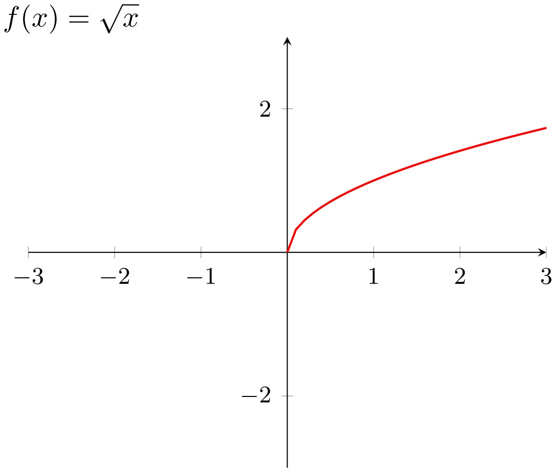
3. (a) $f(x) = -2x + 1$



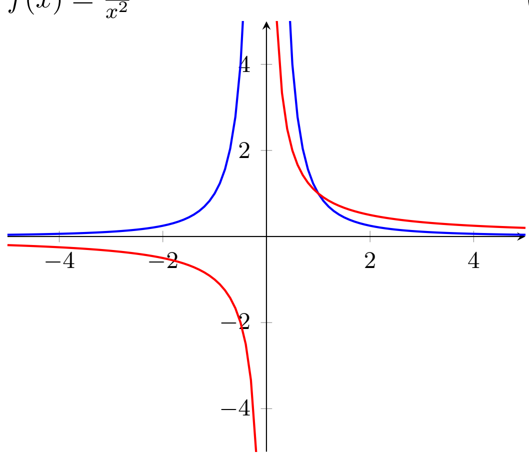
(b) $f(x) = \frac{1}{x}$



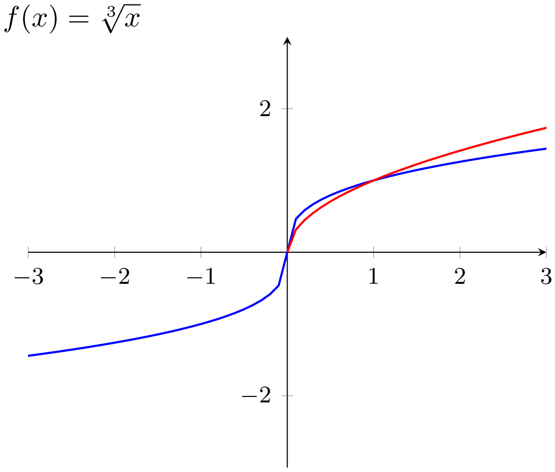
(c) $f(x) = \sqrt{x}$



(d) $f(x) = \frac{1}{x^2}$



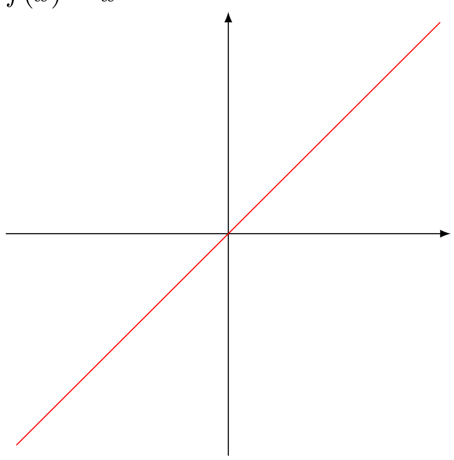
(e) $f(x) = \sqrt[3]{x}$



4. (a) even.
   (b) neither.
   (c) odd.
   (d) even.
   (e) odd.
   (f) odd.

5. They do not have the same graphs.

(a) $f(x) = x$



(b) $g(x) = \frac{x^2}{x}$