Linear functions have constant ***changes****.*
Exponential functions have constant ***percentage changes****!*

1. Chi-Yun decides to open a bank account with an opening deposit of \$1000, and an annual interest rate of 6%.

   - Suppose that the account compounds annually.

     (a) How much money does the account have t years after it is opened?

     Solution: At the end of each year, she earns 6% interest, which has the effect of multiplying her balance by $1 + 0.06$. Therefore, after t years, Chi-Yun has $\$1000(1 + 0.06)^t$.

     (b) How many years does it take for Chi-Yun to have her money doubled?

     Solution: We need to solve $1000 \cdot 1.06^t = 2000$ or $1.06^t = 2$. Hence $t = \log_{1.06} 2 = \frac{\ln 2}{\ln 1.06} \sim 11.9$. Namely it takes about 12 years for the money to double.

     (c) If Chi-Yun wants to have \$1500 after 5 years, how much money should she have deposited at the beginning?

     Solution: We need to solve $P_0 \cdot 1.06^5 = 1500$. Hence $P_0 = 1500/1.06^5 \sim 1120.9$. Namely Chi-Yun should have deposited \$1120.9 to have a total of \$1500 after 5 years.

   - Suppose that the account compounds every 4 months.

     (a) How much money does the account have t years after it is opened?

     Solution: Every 4 months, the account is compounded with an interest rate of $6/3\% = 2\%$. Since there are three 4-month period in a year, after t years, Chi-Yun has $\$1000(1 + 0.02)^{3t}$.

   - Suppose that the account compounds continuously.

     (a) How much money does the account have t years after it is opened?

     Solution: After t years, Chi-Yun has $\$1000e^{0.06t}$.

2. The population in a certain area of the country is increasing. In 1995 the population was 100,000, and by 2015 it was 200,000. If the population has been increasing exponentially and continues to do so, what do you expect the population to be t years after 2015?

Solution: The population is $200000 \cdot 2^{t/20}$, t years after 2015.