

Exponential functions are functions whose percent changes are constant!

1. Erza decides to open a bank account with an opening deposit of \$1000. Suppose that the account earns an annual interest rate of 6%, compounded annually.

(a) How much money does the account have t years after it is opened?

Solution: At the end of each year, he earns 6% interest, which has the effect of multiplying his balance by $1 + 0.06$. Therefore, after t years, Erza has $\$1000(1 + 0.06)^t$.

(b) How many years does it take for Erza to have his money doubled?

Solution: We need to solve $1000 \cdot 1.06^t = 2000$ or $1.06^t = 2$. Hence $t = \log_{1.06} 2 = \frac{\ln 2}{\ln 1.06} \sim 11.9$. Namely it takes about 12 years for the money to double.

(c) If Erza wants to have \$1500 after 5 years, how much money should he have deposited at the beginning?

Solution: We need to solve $P_0 \cdot 1.06^5 = 1500$. Hence $P_0 = 1500/1.06^5 \sim 1120.9$. Namely Erza should have deposited \$1120.9 to have a total of \$1500 after 5 years.

2. The population in a certain area of the country is increasing. In 1995 the population was 100,000, and by 2015 it was 200,000. If the population has been increasing exponentially and continues to do so, what do you expect the population to be t years after 2015?

Solution: The population is $200000 \cdot 2^{t/20}$, t years after 2015.