

Midterm 1

Math 31B-2, Winter 2020

Name:

UID:

Directions—Please read carefully!

- You are allowed **50 minutes** for this exam. Pace yourself, and do not spend too much time on any one problem.
- No notes, books, your own scratch papers, calculators, cell phones, computers, or other electronic aids are allowed.
- In order to receive full credit, you must **show your work or explain your reasoning**; your final answer is less important than the reasoning you used to reach it. Correct answers without work will receive little or no credit.
- Unless otherwise indicated, please simplify your answers.
- You can use the backs of pages as scratch papers, but **only those written in the front of pages** will be graded.
- Please write neatly. Illegible answers will be assumed to be incorrect. **Circle or box your final answer** when relevant.

Good luck!

Question	Points	Score
1	12	
2	18	
3	22	
4	20	
5	28	
Total:	100	

1. You do not need to provide explanation for the following questions.

- (3) (a) Suppose the population in an area grows exponentially. Order the time it takes for the following three scenarios using $<$, $>$, $=$. (eg. $A < B = C$ means B and C take the same time while A takes less time.)
- (A) The population grows from 100 to 150.
 - (B) The population grows from 150 to 200.
 - (C) The population grows from 150 to 225.

- (3) (b) True or False: Let $f(x)$ be an invertible function. If $f(x)$ is increasing, then $f^{-1}(x)$ is also increasing.

- (3) (c) True or False: We can use L'Hôpital's rule to determine the limit

$$\lim_{x \rightarrow 0} \frac{e^x}{x} = \lim_{x \rightarrow 0} \frac{e^x}{1} = 1.$$

- (3) (d) Which integral represents the length of the curve $y = \sin x$ between 0 and $\frac{\pi}{6}$?

$$\int_0^{\frac{\pi}{6}} \sqrt{1 + \sin^2 x} \, dx \quad \int_0^{\frac{\pi}{6}} \sqrt{1 + \cos^2 x} \, dx$$

2. Find the following limits.

(5) (a) $\lim_{x \rightarrow 0} \frac{\tan 4x}{\tan 5x}$

(13) (b) $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x$

3. Let $f(x) = \frac{1}{2}(x + \frac{1}{x})$ be a function with domain $(0, \infty)$ and range $[1, \infty)$.
- (10) (a) Is $f(x)$ an invertible function? If so, explain your reasoning. If not, explain your reasoning, find a restricted domain on which $f(x)$ is invertible, and explain why $f(x)$ is invertible on your restricted domain.
- (12) (b) Write down the formula for the inverse function $f^{-1}(x)$ with domain $[1, \infty)$ and range being the domain/your choice of restricted domain of $f(x)$. (Hint: You will need to use the quadratic formula: the two roots of ax^2+bx+c are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2}$, and choose the correct root accordingly.)

(20) 4. Evaluate the indefinite integral

$$\int \frac{1}{x^3 + x^2 - x - 1} dx.$$

5. Recall that the inverse sine function $\sin^{-1}(x)$ has domain $[-1, 1]$ and range $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

(5) (a) $\cos(\sin^{-1}(x)) = ?$

(8) (b) Use the fact that $\sin^{-1}(x)$ is the inverse function of $\sin(x)$, derive

$$\frac{d}{dx} \sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}.$$

(15) (c) Evaluate the indefinite integral

$$\int \sin^{-1}(x) dx.$$