Math 170E **Homework 7 Solutions**
UCLA, Fall 2026 **due 11/16/2026 at 11:59pm**

Directions. Complete all the problems. Your solutions to the problems should be submitted to Gradescope before the indicated due date above. Please follow rules regarding Gradescope submission as described in the syllabus. If you use any outside source in your solutions, you must cite the source.

Problem 1. Find the mgf of the following random variables.

(a) X which is Bernoulli with parameter p, that is, $X = 1$ with probability p and $X = 0$ with probability $1 - p$.

Final answer. $M_X(t) = 1 - p + pe^t$ for all $t \in \mathbb{R}$.

(b) Y is continuous uniform on an interval $[a, b]$, where $a < b$ are real numbers. *Hint.* Let $M_Y(t)$ denote the mgf of Y. You will need to handle two cases separately: $M_Y(0)$ and $M_Y(t)$ for $t \neq 0$.

Final answer. $M_Y(t) = \dfrac{e^{tb} - e^{ta}}{t(b-a)}$ for $t \neq 0$, and $M_Y(0) = 1$.

Problem 2. Let X and Y be discrete random variables whose joint pmf is

$$f_{X,Y}(x, y) = \frac{x + y}{27}, \qquad x = 1, 2, 3, \quad y = 0, 1, 2.$$

(a) Find the marginal pmf of X.

Final answer. $f_X(x) = \dfrac{3x + 3}{27} = \dfrac{x + 1}{9}$ for $x = 1, 2, 3$ (and 0 otherwise); i.e. $f_X(1) = \frac{2}{9}$, $f_X(2) = \frac{3}{9}$, $f_X(3) = \frac{4}{9}$.

(b) Find $P(X > Y)$.

Final answer. $P(X > Y) = \dfrac{1 + (2 + 3) + (3 + 4 + 5)}{27} = \dfrac{18}{27} = \dfrac{2}{3}$.

(c) Find $P(X + Y \geq 4)$.

Final answer. $P(X+Y \geq 4) = f(2,2)+f(3,1)+f(3,2) = \dfrac{4 + 4 + 5}{27} = \dfrac{13}{27}$.

Problem 3. Two packages are going to be delivered within the next one hour, one red and one blue. Let X be the arrival time of the red package, and let Y be the arrival time of the blue package. Suppose X, Y are jointly uniform on the region

$$D = \{(x, y) \in \mathbb{R}^2 : 0 \le x \le 1, x^2 \le y \le 1\}.$$

(a) Find the joint pdf of X and Y and the marginal pdf of X.

Final answer. The area of D is $\int_0^1 (1 - x^2)\, dx = \frac{2}{3}$, so

$$f_{X,Y}(x, y) = \begin{cases} \frac{3}{2} & \text{if } (x, y) \in D \\ 0 & \text{otherwise,} \end{cases} \qquad f_X(x) = \begin{cases} \frac{3}{2}(1 - x^2) & \text{if } 0 \le x \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

(b) Find the probability that the blue package arrives before the red package.

Final answer. $P(Y < X) = \frac{3}{2} \int_0^1 (x - x^2)\, dx = \frac{3}{2} \cdot \frac{1}{6} = \frac{1}{4}$.

(c) Find $E[XY]$.

Final answer. $\mathbb{E}[XY] = \frac{3}{2} \int_0^1 \int_{x^2}^1 xy\, dy\, dx = \frac{1}{4}$.

Problem 4. Let X and Y be continuous random variables with joint pdf $f_{X,Y}(x, y) = \begin{cases} 3x & \text{if } 0 \le y \le x \le 1 \\ 0 & \text{otherwise.} \end{cases}$

(a) Find the marginal density of X, i.e., compute $f_X(x)$ for all reals x.

Final answer. $f_X(x) = \begin{cases} 3x^2 & \text{if } 0 \le x \le 1 \\ 0 & \text{otherwise.} \end{cases}$

(b) Compute $\mathrm{Var}(X)$.

Final answer. $\mathbb{E}[X] = \frac{3}{4}$, $\mathbb{E}[X^2] = \frac{3}{5}$, so $\mathrm{Var}(X) = \frac{3}{5} - \left(\frac{3}{4}\right)^2 = \frac{3}{80}$.

Problem 5. In lecture, we covered the following fact: let $X_1, \dots, X_n$ be independent, and let M_{X_i} be the mgf of X_i. If M is the mgf of their sum $X_1 + X_2 + \cdots + X_n$, then

$$M(t) = M_{X_1}(t) M_{X_2}(t) \cdots M_{X_n}(t)$$

whenever these are defined at t.

Use this fact, along with your computation in Problem 1(a), to find the mgf of a binomial random variable with parameters $n \in \{1, 2, \dots\}$ and $p \in (0, 1)$.
Final answer. A Binom(n, p) random variable is a sum of n independent Bernoulli(p) random variables, so its mgf is $M(t) = (1 - p + pe^t)^n$ for all $t \in \mathbb{R}$.

Recommended practice. The following exercises from the textbook are not assigned and do not have to be turned in. They are recommended to get more practice.

Section 4.1: 1, 2, 3, 5, 9
Section 4.4: 1, 2, 3, 5, 10, 11

Most of the textbook exercises are very good, so you can always get more practice by working on exercises not mentioned in the above list.