Math 170E **Homework 6 Solutions**
UCLA, Fall 2026 **due 11/9/2026 at 11:59pm**

Directions. Complete all the problems. Your solutions to the problems should be submitted to Gradescope before the indicated due date above. Please follow rules regarding Gradescope submission as described in the syllabus. If you use any outside source in your solutions, you must cite the source.

Problem 1. Let $\lambda > 0$ and let $X \sim \text{Exp}(\lambda)$.

(a) Prove that for any $x, y > 0$,

$$P(X > x + y | X > x) = P(X > y).$$

(This is a memoryless property of the exponential distribution.)

Final answer. Since $P(X > t) = e^{-\lambda t}$ for $t \geq 0$, $P(X > x + y \mid X > x) = \dfrac{P(X > x + y)}{P(X > x)} = \dfrac{e^{-\lambda(x+y)}}{e^{-\lambda x}} = e^{-\lambda y} = P(X > y)$.

(b) Assume now that $\lambda > 1$. Compute $\mathbb{E}[e^X]$ using the rule

$$\mathbb{E}[e^X] = \int_{-\infty}^{\infty} e^x f(x) dx,$$

where f is the pdf of X. Explain why $\mathbb{E}[e^X]$ does not exist when we have $0 < \lambda \leq 1$.

Final answer. $\mathbb{E}[e^X] = \displaystyle\int_0^{\infty} \lambda e^{(1-\lambda)x}\, dx = \frac{\lambda}{\lambda - 1}$. When $0 < \lambda \leq 1$, the integrand $\lambda e^{(1-\lambda)x} \geq \lambda > 0$ does not decay, so the integral diverges to ∞.

Problem 2. Arrivals come according to a Poisson process with rate $\lambda = 3$. The process starts at time $t = 0$.

(a) Let T_{10} be the time of the tenth arrival. Find numbers a and b such that

$$P(T_{10} > 8) = \sum_{x=0}^{a} \frac{b^x}{x!} e^{-b}.$$

Final answer. $a = 9$, $b = 24$.

(b) Let T_3 be the time of the third arrival. Compute $\mathbb{E}\left[\frac{1}{(T_3)^2}\right]$.

Final answer. $\mathbb{E}\left[\dfrac{1}{(T_3)^2}\right] = \dfrac{9}{2}$.

Problem 3. Let $X \sim N(\mu, \sigma^2)$ (where $\sigma^2 > 0$) and let a, b be constants with $a \neq 0$. Show that $Y = aX + b \sim N(a\mu + b, a^2\sigma^2)$.

Hint. Write down a formula for the cdf $P(Y \leq y)$ of Y and use the fundamental theorem of calculus to take a derivative.

Final answer. Differentiating the cdf of Y (separately for $a > 0$ and $a < 0$) gives

$$f_Y(y) = \frac{1}{|a|} f_X\left(\frac{y-b}{a}\right) = \frac{1}{|a|\sigma\sqrt{2\pi}}\, e^{-(y-a\mu-b)^2/(2a^2\sigma^2)},$$

which is the pdf of $N(a\mu + b,\, a^2\sigma^2)$.

Problem 4. The serum zinc level X in micrograms per deciliter for males between ages 15 and 17 has distribution that is approximately normal with $\mu = 90$ and $\sigma = 15$.

(a) Compute $P(|X - 90| > 20)$.

Final answer. $P(|X-90| > 20) = 2\,(1 - \Phi(4/3)) \approx 0.1824$. (Rounding to $z = 1.33$ in a table gives ≈ 0.1836.)

(b) Compute the conditional probability $P(X > 114 | X > 99)$.

Final answer. $P(X > 114 \mid X > 99) = \dfrac{1 - \Phi(1.6)}{1 - \Phi(0.6)} \approx \dfrac{0.0548}{0.2743} \approx 0.1998.$

Problem 5. Let X be a discrete random variable whose MGF is of the form

$$M_X(s) = a + be^{2s} + ce^{4s}$$

Moreover, assume $\mathbb{E}[X] = 3$ and $\mathrm{Var}(X) = 2$. Find a, b, and c, and find the pmf of X.

Final answer. $a = \dfrac{1}{8}$, $b = \dfrac{1}{4}$, $c = \dfrac{5}{8}$. The pmf of X is

$$p_X(x) = \begin{cases} 1/8 & \text{if } x = 0 \\ 1/4 & \text{if } x = 2 \\ 5/8 & \text{if } x = 4 \\ 0 & \text{otherwise.} \end{cases}$$

Recommended practice. The following exercises from the textbook are not assigned and do not have to be turned in. They are recommended to get more practice.

Section 3.2: 2, 12, 16
Section 3.3: 1, 2, 3, 11

Most of the textbook exercises are very good, so you can always get more practice by working on exercises not mentioned in the above list.