### COMPLEX ANALYSIS

# by T.W. Gamelin

# Springer-Verlag, UTM Series

Changes for the second printing (compiled in March, 2003)

## CHAPTER I

p.8, l.15: Change "imiginary" to "imaginary" (spelling).

p.8, l.18: Change " nth " to " nth " (boldface italic en).

p.10, Ex.6(b): Change "= 0," to "= 0 if  $n \ge 2$ ," (insert "if  $n \ge 2$ " after the zero).

p.18, l.8: Change second subscript from "1" to "2" (should be " $f_1(w)$  and  $f_2(w)$ ").

p.24, l.-1: Change " $i \log(-i)$ " to " $-i \log i$ " (should be " $e^{-i \log i}$ ").

p.32, Ex.5, l.-1 and l.-2: Change "horizontal strip  $\{-\pi < \text{Im}\, w < \pi\}$ " to "vertical strip  $\{|\text{Re}\, w| < \pi/2\}$ "

#### CHAPTER II

p.40, Ex.6(a): Exercise 6(a) should read:

The sequence  $\binom{\alpha}{n}$  is bounded if and only if  $\operatorname{Re} \alpha \geq -1$ .

p.49, l.12: Change " $e^z$ " to " $e^{az}$ " (insert an "a" in the exponent)

p.54, Ex.9, l.3: Change "  $\leq$  " to " < "

p.61, Ex.2, l.2: Change "one of the figures in this section" to "a figure in Section I.6"

### CHAPTER III

p.96, Ex.2: In the displayed equation, change "  $\beta \mathbf{u}_{\theta}$  " to

$$\frac{\beta}{r}\mathbf{u}_{\theta}$$

p.97, Ex.4, l.-3: Change "are circles" to "are arcs of circles"

p.97, Ex.8: Change the displayed equation to read:

$$\mathbf{V}(r,\theta) = \frac{1}{r}(-\mathbf{u}_r + \mathbf{u}_\theta).$$

p.101, Ex.6, l.-1: Change "the preceding exercise and an appropriate conformal map " to "Exercise 5 and the conformal map from Exercise 4 "

### CHAPTER IV

```
p.106, Ex.5, l.2: Change "3" to "1"
```

p.113, l.-6: Change "
$$f(z)$$
" to " $f(w)$ "

p.116, l.6: Change "
$$|z| = 5$$
" to " $|z| = 2$ "

p.117, Ex.1(h): Change "3" to "2" (should be "
$$|z-1|=2$$
")

- p.120, l.5: Delete "as indicated in the figure"
- p.122, Ex.2, l.5: Change "x + +iy" to "x + iy" (delete a plus sign)
- p.123, Ex.4, l.1: Change " and let " to " let " (delete " and ")
- p.128, Ex.3: Change " $J_f$ " to "det  $J_f$ " (add "det" in roman font)

## CHAPTER V

- p.133, Ex.4, l.2: Change " $(-1)^k$ " to " $(-1)^{k+1}$ " (change "k" to "k+1" in the exponent)
- p.133, Ex.4, l.4: Change "  $S_1 < S_3 < S_5 < \cdots$  and  $S_2 > S_4 > S_6 > \cdots$  " to "  $S_2 < S_4 < S_6 < \cdots < S_5 < S_3 < S_1$  "
  - p.136, l.-1: Change "k+1" to "m+1" (denominator should be  $(s-r)^{m+1}$ )
  - p.137, l.2: Change "k+1" to "m+1" (denominator should be  $(s-r)^{m+1}$ )
- p.138, Ex.11, l.3: Change "  $z \in D$  " to "  $z \in E$  " (change the first cap dee in the line to cap ee)
  - p.143, Ex.1(b): Change "6" to " $6^k$ " (raise "6" to the kth power)
  - p.154, Ex.4, l.4: Change "five" to "three"
- p.156, l.-12: Change " f(z) has distance " to "  $z_0$  has distance " (change the second " f(z)" in the line to " $z_0$ ")
  - p.163, Ex.6, l.-1: Change "  $1 \leq \operatorname{Re} z < 1 + \varepsilon$  " to "  $|z-1| < \varepsilon$  "

p.170, Ex.3, l.-3: In the displayed formula, change " $i(n\theta - z\sin\theta)$ " to " $i(z\sin\theta - n\theta)$ " (change the sign of the exponent). The formula is correct as it stands, but it led to confusion.

p.171, Ex.7, l.-1: Add "(See Exercise III.3.4.)" at the end.

p.174, l.8: Change the displayed equation to read:

$$f_1(z) = \frac{1}{z} - \frac{1}{z+\pi} - \frac{1}{z-\pi},$$

(This amounts to changing the sign of the expression for  $f_1(z)$ .)

p.174, l.13: Delete "the first two"

p.174, l.14: Change the displayed equation to read:

$$f_1(z) = \frac{1}{z} - \frac{2z}{z^2 - \pi^2} = \frac{1}{z} - \frac{2}{z} \sum_{k=0}^{\infty} \frac{\pi^{2k}}{z^{2k}} = -\frac{1}{z} - \sum_{k=1}^{\infty} \frac{2\pi^{2k}}{z^{2k+1}}.$$

(This amounts to changing the sign of the expression for  $f_1(z)$ .)

p.174, l.15: Change " $a_{-1}=1$  and  $a_{-3}=2\pi^2$ " to " $a_{-1}=-1$  and  $a_{-3}=-2\pi^2$ " (insert two minus signs)

p.178, Ex.15, l.3: Change "|z|" to "z" (delete vertical bars)

### CHAPTER VII

p.202, Ex.8: Change the displayed equation to read:

$$\int_{-\infty}^{\infty} \frac{\cos x}{(1+x^2)^2} dx = \frac{\pi}{e}.$$

(Replace the right-hand side by pi over e.)

p.205, Ex.5: The fraction in the integral should read:

$$\frac{1-r^2}{1-2r\cos\theta+r^2}$$

(change " 1 " to "  $1-r^2$  " in the numerator of the fraction)

p.206: In the displayed figure, change "  $\Gamma_{\varepsilon}$  " to "  $\gamma_{\varepsilon}$  " (change cap gamma to lower case gamma)

p.210: In the displayed figure, change " $\Gamma_{\varepsilon}$ " to " $\gamma_{\delta}$ " (change cap gamma sub epsilon to lower case gamma sub delta)

p.216, l.-10: Change " $R \sin \theta$ " to " $-R \sin \theta$ " (insert minus sign in the exponent)

p.216, l.-1: Change " 1/R " to " R " (delete " 1/ " in the upper limit of the integral)

### CHAPTER VIII

p.225, l.-9: After " $\gamma$  in D" insert "providing there are no zeros or poles on the path"

p.231, l.-5: Change "integral" to "expression"

p.231, l.-4: Change "integral" to "expression"

p.234, l.2: Delete " =  $N_0$ " (identity should be "  $N(w_0) = 1$ ")

p.237, l.-6: Change " af " to " of " (spelling)

p.240, Ex.14(b): Change part (b) of the exercise to read:

(b) Glue together branch cuts to form an *m*-sheeted (possibly disconnected) surface over the punctured disk  $\{0 < |w| < \delta\}$  on which the branches  $z_j(w)$  determine a continuous function.

p.241, Ex.15(b), l.2: Change " $AP_1 + BP_2$ " to " $AP_0 + BP_1$ " (change subscripts)

p.242, Ex.15(e), l.2: Change "then the coefficients of each of the irreducible factors of P(z,w)" to "then the irreducible factors of P(z,w) can be chosen so that their coefficients"

p.242, Ex.15(g), l.1: Change "14(a)-(d)" to "14(a)-(c)" (change "d" to "c")

p.242, l.14: Change "Thus" to "If  $\gamma$  is piecewise smooth,"

p.242, l.-5: Delete ", the integrals in (6.1) are zero, and " (should read "Consequently  $W(\gamma, z_0) = 0$ .")

p.244, l.5: Change the displayed equation so that the right-hand side reads

$$= - \int_{\partial U} W(\gamma, \zeta) f(\zeta) d\zeta.$$

(Delete the 1 over  $2\pi i$  before the last integral, but leave the minus sign.)

p.245, Ex.5, l.1: Change "a closed curve" to "a piecewise smooth closed curve" (insert "piecewise smooth")

p.252, l.-14: Change "are paths" to "are closed paths" (insert "closed")

p.252, l.-4: Change "It turns out that if" to "If"

p.253: In the displayed figure, change "S" to "s" (lower case ess)

# CHAPTER IX

p.265, Ex.2: Change "  $(3+z^2)/(1+3z^2)$  " to "  $(1+3z^2)/(3+z^2)$  " (the fraction is upside down)

p.278, l.17: Change " $e^{i\theta}$ " to " $re^{i\theta}$ " (insert "r")

p.280, Ex.8(a), l.-2: In the displayed equation, change " $z=e^{i\theta}$ " to " $z=re^{i\theta}$ "

p.282, Ex.2-5: Replace Exercises 2, 3, 4, and 5 by the following Exercises 2, 3, and 4:

- 2. Assume u(x,y) is a twice continuously differentiable function on a domain D.
- (a) For  $(x_0, y_0) \in D$ , let  $A_{\varepsilon}(x_0, y_0)$  be the average of u(x, y) on the circle centered at  $(x_0, y_0)$  of radius  $\varepsilon$ . Show that

$$\lim_{\varepsilon \to 0} \frac{A_{\varepsilon}(x_0, y_0) - u(x_0, y_0)}{\varepsilon^2} = \frac{1}{4} \Delta u(x_0, y_0),$$

where  $\Delta$  is the Laplacian operator (Section II.5).

(b) Let  $B_{\varepsilon}(x_0, y_0)$  be the area average of u(x, y) on the disk centered at  $(x_0, y_0)$  of radius  $\varepsilon$ . Show that

$$\lim_{\varepsilon \to 0} \frac{B_{\varepsilon}(x_0, y_0) - u(x_0, y_0)}{\varepsilon^2} = \frac{1}{8} \Delta u(x_0, y_0).$$

- 3. For fixed  $\rho > 0$ , define  $h(z) = e^{i\rho \text{Im } z}$ . Show that if  $\rho$  is a zero of the Bessel function  $J_1(z)$ , then  $\int_{\gamma} h(z) dz = 0$  for all circles  $\gamma$  of radius 1. Suggestion. See the Schlömilch formula (Exercise VI.1.3).
- 4. Suppose that  $r_1, r_2 > 0$  are such that  $r_2/r_1$  is a quotient of two positive zeros of the Bessel function  $J_1(z)$ . Show that there is a continuous function g(z) on the complex plane such that  $\int_{\gamma} g(z) dz = 0$  for all circles of radius  $r_1$  and for all circles of radius  $r_2$ , yet g(z) is not analytic. Use the preceding exercise together with the fact (easily derived from the differential equation in Section V.4) that  $J_1(z)$  has zeros on the positive real axis. Remark. The condition is sharp. If  $r_2/r_1$  is not the quotient of two positive zeros of the Bessel function  $J_1(z)$ , then any continuous function f(z) on the complex plane such that  $\int_{\gamma} f(z) dz = 0$  for all circles of radius  $r_1$  and  $r_2$  is analytic, by a theorem of L. Zalcman. There is an analogous result for harmonic functions and the mean value property. According to a theorem of J. Delsarte, if  $r_1, r_2 > 0$  are such that  $r_2/r_1$  is not the quotient of two complex numbers for which  $J_0(z) = 1$ , then any continuous function on the complex plane that has the mean value property for circles of radius  $r_1$  and  $r_2$  is harmonic.

p.288, Ex.10(d): Make changes in lines 1 and 3, so that Exercise 10(d) reads as follows: Show that if  $\varphi(z)$  is a solution of Schröder's equation (analytic at 0 and satisfying  $\varphi(0) = 0$ ,  $\varphi'(0) = 1$ ), and if  $\varphi(x)$  is real when x is real, then  $\varphi(z)$  maps the angle between the positive real axis and  $\gamma$  to the angle between the straight line segments at angles 0 and  $\theta_0$ .

### CHAPTER XI

- p.295, l.7: Change "domain" to "domain in the plane" (insert "in the plane")
- p.295, l.8: Change "the complement" to "it"
- p.296, l.11: Change "on the side" to "on one side"
- p.296, l.12: Change "on other side" to "on the other side"
- p.297: In the displayed figure, change " $\alpha\pi$ " to " $\pi\alpha$ " and reverse the direction of the arrow to the left of the alpha so that the arrow points up rather than down
- p.298, l.-13: In equation (3.2), change " 2 " to "  $\alpha+1$  " (change fraction " 2 over zee " to "  $\alpha+1$  over zee ")
  - p.299, l.-11: Change "do not" to "may not"
- p.300: In the displayed figure, change " $\alpha_0$ " to " $\alpha_1$ ", change " $\alpha_1$ " to " $\alpha_2$ ", and change " $\alpha_2$ " to " $\alpha_3$ " (increase each subscript by one)
  - p.304, l.-3: Change "(4.3)" to "(3.4)"
  - p.308, l.10: Change "XIV" to "XII"
- p.311, Ex.9(a), l.1: Change "B" to script cap "F". Use the same font for script cap eff as later in the same line.
  - p.313, l.8: Insert " $\varphi(z_0) = 0$ ," before "and"
  - p.313, l.15: Change " $(z_0|$ " to " $(z_0)|$ " (insert right parenthesis after subscript zero)
  - p.313, l.17: Change "give another" to "sketch a different"

### CHAPTER XII

```
p.316, l.-1: Change " no zeros " to " no zeros near z_0 "
```

p.319, Ex.3, l.1: Change "
$$z/(z+\varepsilon)$$
" to " $f_{\varepsilon}(z)=z/(z+\varepsilon)$ " (insert " $f_{\varepsilon}(z)=$ ")

p.319, Ex.4, l.1: Change " 
$$1/(z+\varepsilon)$$
 " to "  $z^3/(z+\varepsilon)$  " (change " 1 " to "  $z^3$  ")

- p.319, Ex.8, l.3: Change "VIII.4.2" to "VIII.4.5"
- p.320, l.-3: Change "h(z)" to " $h(\zeta)$ " (change zee to zeta)
- p.324, Ex.11, l.7: Change " $\psi(z)$ " to " $\psi(w)$ " (change zee to double-u)
- p.331, Ex.14, l.2: Change "cycle of " to "cycle of " (insert space)
- p.336, l.14: Change "compact set subset" to "compact subset" (delete "set")
- p.336, Ex.2, l.1: Insert "whose Julia set is connected" after "d > 2"
- p.337, Ex.3, l.-1: Change "m" to "m+1" (should read " $\zeta^{m+1}$ ")
- p.341, Ex.10, l.1: Change "W" to "V"
- p.341, Ex.10, l.3: Change "W" to "V"
- p.341, Ex.10, l.4: Change "W" to "V"

#### CHAPTER XIII

```
p.345, l.2: Change "\Delta_i" to "D_i"
```

- p.351, l.3: In the displayed equation, change " $\sin^2(\pi z)$ " to " $\sin^2(\pi \zeta)$ " (change the zee to a zeta)
  - p.352, Ex.7, l.2: Change "double" to "simple"
  - p.352, Ex.7, l.3: Delete " $1/(z \log n)^2 +$ "
  - p.353, l.13: Change "are" to "is"
- pp.357-8, Ex.16(a), l.3-4 (bottom line of p.357 and top line of p.358): Change "uniformly on compact subsets of " to "normally on "
  - p.359, l.-14: In the displayed equation, change " z " to "  $\zeta$  "

#### CHAPTER XIV

```
p.364, l.8: Change "XIII.3.1" to "XIII.4.1" (change "3" to "4")
```

- p.367, l.3: Change " k=1 " to " k=0 " beneath the summation sign on the left-hand side of the equation.
  - p.368, l.2: Change "1.3(c)" to "1.3" (delete "(c)")
  - p.368, l.3: Change "2n-1" to "1-2n" (should read " $2^{1-2n}$ ")
  - p.372, l.6: Change "above" to "below"

p.373, l.-12: In the displayed equation, change the "1" just to the left of the big square bracket to "0". (The residue is evaluated at 0.)

p.373, l.-3: Change "above" to "below"

p.374, l.2: Change "2n + 1/2" to "2n + 1" (delete "2")

p.375, Ex.2, l.2-3: Change "that is symmetric with respect to the line  $\{\text{Re }s=\frac{1}{2}\}$ , that is, "to "that satisfies" (replace the passage from "is" to "is," by "satisfies")

p.378, l.-7: In right-hand term of the displayed formula, change " |s| " to "  $C\sigma$  ". Add a period at the end of the line.

p.378, l.-6: Delete this line of displayed formula.

p.378, l.-5: Change "in (4.5), we obtain a telescoping series, " to "in (4.5) and sum, we obtain "

p.378, l.-4: Change the middle term of the displayed formula to

$$\varepsilon_m \left( 2 + C\sigma \int_m^n r^{-\sigma - 1} dr \right)$$

p.378, l.-3: Change "we obtain uniform convergence." to "the series converges uniformly in the sector, and consequently it converges pointwise in the open half-plane. This completes the proof of the theorem."

p.379, Ex.1, l.1: Change " (4.5) that  $\zeta(\sigma)>0$  " to " (4.6) that  $\zeta(\sigma)<0$  " (change " 5 " to " 6 " and change " > " to " < ")

p.379, Ex.5, l.1: Change "Mobius" to "Möbius" (add umlaut)

p.379, Ex.5, l.2: Change "n" to "n" (change font)

p.380, Ex.6: Replace Exercise 6 by the following exercise:

6. The **Dirichlet convolution** of the sequences  $\{a_n\}_{n=1}^{\infty}$  and  $\{b_n\}_{n=1}^{\infty}$  is the sequence  $\{c_n\}_{n=1}^{\infty}$  defined by

$$c_n = \sum_{d|n} a_d b_{n/d} \,.$$

Show that if the Dirichlet series  $\sum a_n n^{-s}$  and  $\sum b_n n^{-s}$  converge in some half-plane to f(s) and g(s) respectively, then the Dirichlet series  $\sum c_n n^{-s}$  converges in some half-plane to f(s)g(s).

p.380, Ex.9, l.2: Delete "and determine where they are valid"

p.380, Ex.9: Add the following line after the displayed equations: " *Hint*. Use Exercise 4."

p.381, Ex.11(a), l.1: Insert space after "and"

p.383, l.-6: Change "treat" to "treats"

p.390, l.9: Change "give" to "sketch"

p.390: In the displayed figure, replace the "t" with an arrow over it by " $\mathbf{t}$ ", and replace the "n" with an arrow over it by " $\mathbf{n}$ " (should be boldface tee and boldface en, with no arrows)

p.393, Ex.8, l.3-4: The identity on lines 3 and 4 should read:

$$\rho \int_{0}^{2\pi} h(\rho e^{i\theta}) d\theta = \sigma \int_{0}^{2\pi} h(\sigma e^{i\theta}) d\theta$$

(insert Greek rho before the first integral and Greek sigma before the second integral)

p.394, l.-8: Change "8" to "9" (should refer to "Exercise 9")

p.405-6, Ex.4-5: Replace Exercises 4 and 5 by the following exercise:

4. Let D be a bounded domain, and let h(z) be a continuous function on  $\partial D$ . Let  $\{D_m\}$  be a sequence of smoothly bounded domains that increase to D. Let u(z) be a continuous extension of h(z) to  $D \cup \partial D$ , and let  $u_m(z)$  be the harmonic extension of  $u|_{\partial D_m}$  to  $D_m$ . (a) Show that  $u_m$  converges uniformly on compact subsets of D to a harmonic function Wh on D. Hint. If u(z) is smooth, represent u(z) as the difference of two subharmonic functions, as in Exercise 2.6. (b) Show that Wh depends only on h, and not on the particular extension u of h to D nor on the sequence  $\{D_m\}$ . (c) Show that W is linear, that is,  $W(ah_1 + bh_2) = aW(h_1) + bW(h_2)$ . (d) Show that  $\tilde{h} \leq Wh$ , where  $\tilde{h}$  is the Perron solution to the Dirichlet problem. Remark. The harmonic function Wh is the Wiener solution to the Dirichlet problem with boundary function h. It can be shown that the Wiener solution coincides with the Perron solution.

p.406, l.7: Change "give" to "sketch"

p.406, l.17: Change "We construct a subharmonic barrier at  $\zeta_0$ ." to "Proofsketch."

p.406, l.21-25: Replace these five lines by the following:

and Re f(z) < -1 for  $z \in D \cap D_0$ . Define h(z) = Re(1/f(z)) for  $z \in D \cap D_0$ . Then h(z) < 0. Since Re  $f(z) \to -\infty$  as  $z \to \zeta_0$ ,  $h(z) \to 0$  as  $z \to \zeta_0$ , and h(z) is "almost" a barrier at  $\zeta_0$ . The difficulty is that h(z) might tend to 0 at other boundary points of  $D \cap D_0$ . The proof is completed by following the procedure in the proof of Bouligand's lemma (pp. 8-9 of Tsuji's book; see the references), in which a genuine barrier at  $\zeta_0$  is constructed from h(z).

p.407, l.8: Change "  $0<\varepsilon<|w|$  " to "  $|w|<1-\varepsilon$  "

p.414. l.-13: Change " $\log(z-\zeta)$ " to " $\log|z-\zeta|$ " (replace parentheses by vertical bars)

p.416, Ex.11, l.1-2: Change "the point at  $\infty$ " to "the exterior of a disk"

### CHAPTER XVI

```
p.418, l.-15: Change " near sighted " to " near
sighted " (it's one word)
```

p.418, l.-2: Change "
$$\pi$$
" to " $z$ " (should read " $z_{\alpha}^{-1}(D_0)$ ")

p.418, l.-1: Change " 
$$\pi$$
 " to "  $z$  " (should read "  $z_{\beta}^{-1}(D_1)$  ")

p.423, Ex.3(b): Part (b) of the exercise should read: "(b) if the pole is not simple, a coordinate can be chosen with respect to which f(p) has residue zero at  $p_0$ ."

```
p.423, Ex.6, l.4: Change " \Delta " to " \mathbb D "
```

p.425, Ex.13, l.2: Change script cap ell subscript tau to " $L_{\tau}$ " (the cap ell should be the same font as in the line above it)

p.426, Ex.14(d), lines 2 and 4: Change " z-d/c " to " z+d/c " (change minus sign to plus sign twice)

```
p.427, l.-13: Change " is " to " its "
```

p.428, l.-15: Delete " given by " 
$$\,$$

p.430, l.13-14: Change the sentence starting the paragraph at line 13 to read as follows: As the upper envelope of a Perron family, g(p,q) is harmonic, and further  $g(p,q) \ge 0$ .

```
p.433, Ex.2, l.1: Change "onto" to "to"
```

p.434, l.-9: Change "vectors" to "direction"

p.436, lines -10 and -9: Change "  $z_1(q)$  " to "  $z_1(p)$  ", once on each line

p.444, l.19: Change "unit" to "union"

p.444, Ex.3, l.2: Delete "one-to-one and" (should read "then  $\varphi$  is onto.")

p.444, Ex.3, l.3: Delete " Remark. Thus  $\varphi$  is a covering transformation." (delete the entire line)

p.446, Ex.10, l.4: Change "for" to "from"

#### HINTS AND SOLUTIONS FOR SELECTED EXERCISES

```
p.447, I.1, Ex.1(d): Should be "[-1,1]"
   p.448, I.3, Ex.6(b): Change "Y_1 - X_2" to "Y_1 - Y_2"
   p.449, II.2, Ex.4: Should read:
4. Use (f(z + \Delta z) - f(z))/\Delta z \approx 2az + b\bar{z} + (bz + 2c\bar{z})\overline{\Delta z}/\Delta z.
   p.451, II.7, Ex.2: Change "Unit circle" to "Circle", change "unit disk" to "disk",
and change "unit circle" to "circle, that is, with slope = 1"
   p.451, II.7, Ex.8: Change "by \alpha\delta - \beta\gamma" to "by the square root of \alpha\delta - \beta\gamma"
   p.451, III.2, Ex.1(b): Change "2x^2" to "2x^3"
   p.452, III.5, Ex.6: Change "(z+1)^{\varepsilon}f" to "(z+1)^{-\varepsilon}f" (insert minus sign)
   p.452, III.5, Ex.9: Change " 0 < z < \delta " to " 0 < |z| < \delta"
   p.452, III.6, Ex.1: Change " (2-1)z " to " (2-i)z "
   p.453, IV.1, Ex.3(b): Change "m" to "m+1". Should be "2\pi R^{m+1}"
   p.453, IV.1, Ex.3(c):Change "R^m" to "R^2"
   p.453, IV.4, Ex.1(h): Should be "-\pi i/2 + \pi i/4e^2"
   p.454, V.2, Ex.2: Change "[1, 1 - \varepsilon]" to "[0, 1 - \varepsilon]"
   p.456, VI.2, Ex.3(a), l.2: Change "part" to "parts", and change sign three times.
Second line should read:
parts -1/(z \pm \pi/2). If f_1(z) = -1/(z - \pi/2) - 1/(z + \pi/2), then f_0(z) =
   p.457, VII.1, Ex.3(f): Change " -1 " to " -2/\pi" and change " -2\pi i " to " -4i"
   p.459, VIII.2, Ex.6(b): Should be "First and fourth quadrants."
   p.462, IX.2, Ex.12(c): Should be "for (c) there are two, the identity f(z) = z, and
```

f(z) = -2/z. "

p.466, XV.2, Ex.5(a): Should be "IV.8.7"

# LIST OF SYMBOLS

p.471: add the following symbols:

$\partial$						(0)	
$\frac{\overline{\partial z}}{\partial z}$	partial	derivative	with	respect	to $z$	(Section	IV.8)

 $\frac{\partial}{\partial \bar{z}}$  partial derivative with respect to  $\bar{z}$  (Section IV.8)

 $f^{\sharp}$  spherical derivative (Section XII.1)

# **INDEX**

p.476: Between the "Neumann problem" and "normal convergence of meromorphic functions", insert "normal convergence of analytic functions, 137"

p.477: Between "radius of convergence" and "ratio test", insert "Radó's theorem, 432"