1. Suppose we have two random variables, X and Y, whose joint density is $f_{X,Y}(x,y)$. Recall that this means that for any set A in the plane,

$$\mathbb{P}((X,Y) \in A) = \iint_A f_{X,Y}(x,y)\ dydx$$

and that this means, for any function g,

$$\mathbb{E}[g(X,Y)] = \iint_A g(x,y) f_{X,Y}(x,y)\ dydx$$

Set up integrals (or expressions involving several integrals) for the following:

$$\mathbb{P}(X^2 + Y^2 \leq 1)$$
$$\mathrm{Var}[X+Y]$$
$$\mathbb{E}[\sin(X)|X+Y>4]$$

2. Show that if X and Y are independent random variables with densities f_X and f_Y, then

$$\mathbb{E}[XY] = \mathbb{E}[X]\mathbb{E}[Y]$$

so long as $\mathbb{E}[XY]$ is finite. (Hint: Write out the left-hand side in terms of the joint density).

3. Suppose we have independent random variables X and Y such that X is exponentially distributed with parameter 3 and Y is uniformly distributed on $[0,2]$. Compute

$$\mathbb{P}(X > Y)$$
$$\mathbb{E}[\max(X,Y)]$$

(or just set up integrals that could be solved to find the answers).