Review - I will do #1 and #2 on the board and then answer questions.

1. Suppose we have a random variable with PDF

$$f_X(x) = \frac{C}{1+x^2}$$

for some constant C.

- Find C.
- Find $\mathbb{E}[X]$ and $\mathrm{Var}(X)$.

2. Let T be the triangle in the plane defined by the relations $0 \le y \le x \le 1$. Consider the following ways of picking a point (X, Y) 'uniformly' on T:

- First pick X uniformly on $[0, 1]$. Then pick Y uniformly on $[0, X]$.
- The usual uniform distribution on T.
- First pick Y uniformly on $[0, 1]$. Then pick X uniformly on $[Y, 1]$.

Show that all of these methods give different values of $\mathbb{E}[Y]$, so they are not equivalent, even though they are all in a sense uniform. This kind of inconsistency was a major problem before the formalization of probability theory (see, for instance, `https://en.wikipedia.org/wiki/Borel%E2%80%93Kolmogorov_paradox` or `https://en.wikipedia.org/wiki/Bertrand_paradox_(probability)` for good examples)

3. True-false questions. For each of the following statements, determine whether it is true or false, and have a rough idea of why.

(a) For any random variables X and Y, $\mathbb{E}[XY] = \mathbb{E}[X] \cdot \mathbb{E}[Y]$.

(b) There exists a random variable which is uniformly distributed on the integers.

(c) There exists a random variable which is uniformly distributed on $\mathbb{R}$.

(d) If a random variable does not have expectation, it does not have variance.

(e) There exist random variables which are neither continuous nor discrete.

(f) A random variable has density if and only if the CDF is differentiable.

(g) If a random variable is symmetrically distributed about c, then it must have expectation c.

(h) If the probability of an event is zero, then that event must be empty.

(i) If a function f is nonnegative and $\int_{\mathbb{R}} f = 1$, then f is the PDF of some random variable X.