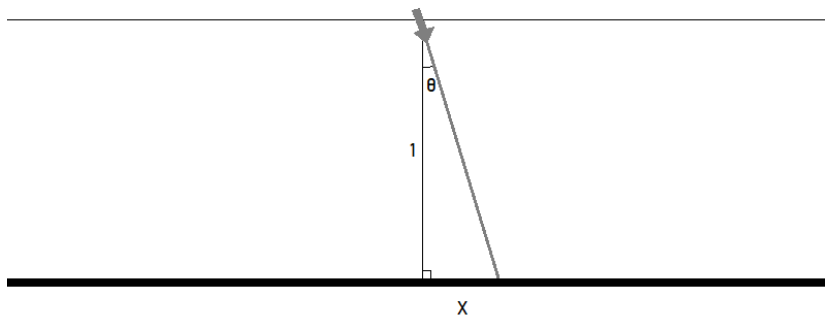


1. Show that the variance of a geometric random variable with parameter  $p$  is  $\frac{1-p}{p^2}$ . What does this mean if  $p \approx 0$  or  $p \approx 1$ ? (I'm going to do this example for the class; I'm just writing it here for convenience)
2. Suppose we have a flashlight at the point  $(0, 1)$  and a wall on the  $x$ -axis. We give it a spin and wait for it to stop at a random angle  $\Theta$  from the  $y$ -axis. We can take  $\Theta$  to be distributed uniformly on  $(-\pi/2, \pi/2)$ . Let  $X$  be the point on the  $x$ -axis where the beam of light hits. (See the picture).



- (a) What is the relationship between  $\Theta$  and  $X$ ? (This is a trigonometry problem).
- (b) For any real  $x$ , find  $F_X(x) = \mathbb{P}(X \leq x)$ ? (This is called the *cumulative distribution function* of  $X$ , and completely describes the distribution of  $X$ ).
- (c) Use  $F_X$  to find the density  $f_X$  of  $X$ . (Hint: it is of the form  $\frac{C}{1+x^2}$  for some constant  $C$ ).
- (d) Show that  $\mathbb{E}X \neq 0$  even though the distribution of  $X$  is symmetric about zero.