1. **Probability mass functions with infinite support**. Suppose we roll a fair six-sided die until we get a 3.

   (a) Describe the sample space Ω of this experiment.

   (b) Pick a representative element of Ω and find the probability associated with this element.

   (c) Let X be the number of rolls we end up making. Find the probability mass function of X.

   (d) Show that your probability mass function obeys the axioms of probability mass functions.

2. **Probability mass functions must decay**. Suppose we have a random variable X with probability mass function f. Show that

$$\lim_{x\to\infty} f(x) = 0.$$

3. **Independence of random variables**. As mentioned in class, the concept of *independence of random variables* is central to what we'll be doing for the rest of 170A.

   (a) Suppose we want to say that random variables X and Y are independent. What would you like this to mean?

   (b) Formalize what you were saying in part (a).

   (c) To every event E we can assign a natural choice of random variable:

$$\mathbb{1}_E(\omega) = \begin{cases} 1 & \omega \in E \\ 0 & \omega \notin E \end{cases}$$

   This is called the *indicator random variable* corresponding to E. Show that your definition satisfies

$$\mathbb{1}_E \text{ and } \mathbb{1}_F \text{ independent as RVs} \iff E \text{ and } F \text{ independent as events}$$