1. **Independence of complements**. Show that if events A and B are independent, so are A^c and B^c.

2. **Self-independence**. Under what conditions is an event A independent of itself?

3. **All of the independence conditions matter**. We've already seen that, for three events A, B, and C, pairwise independence does not imply independence. (Can you construct a simple example?)

   Say we pick a number uniformly at random from $\{1, 2, \ldots, 8\}$. Define three events as follows:

   $$\begin{aligned} A &= \{\text{an even number is chosen}\} \\ B &= \{\text{a prime number is chosen}\} \\ C &= \{\text{a number } \leq 4 \text{ is chosen}\} \end{aligned}$$

   Show that we have $\mathbb{P}(A \cup B \cup C) = \mathbb{P}(A)\mathbb{P}(B)\mathbb{P}(C)$, and we have that two of the possible pairs of events are independent. Nevertheless, show that the three events are not independent.

4. **Counting**. To show that n events are independent, how many conditions do we have to check? And of these conditions, how many are nontrivial (that is, not true for every possible set of n events)?

5. **Challenge**. (This is a problem which is not necessarily relevant to the class; it's here in case you finish the rest of the problems and want something fun to do. It relies on some ideas which are not 170A-type things.)

   (a) Suppose we roll 2 fair die. Let A be the event that the sum of the two die is 12. Find all events which are independent of A.

   (b) Let B be the event that the sum of the two die is 8. Find all events which are independent of B.

   (c) Now suppose we roll 3 die. Find a sum for the three die that does the same thing as what happened in part (b).