1. **An example of a probability law**. We'll construct the probability law corresponding to rolling two (fair) six-sided dice. Let Ω be the set of ordered pairs with each coordinate an integer from 1 to 6. So

$$\Omega = \{(1,1),(1,2),\cdots,(6,6)\}.$$

Now, we can define a function $\mathbb{P}$ defined on subsets of Ω as follows:

$$\mathbb{P}(S) = \frac{|S|}{36} \qquad \text{for any } S \subseteq \Omega.$$

(a) Verify that $\mathbb{P}$ is a valid probability law.

(b) Define the event A to be the subset of Ω where both of the die come up with even numbers. And let the event B be the subset of Ω where the sum of the die rolls is 8. Write down what A and B are. Find $\mathbb{P}(A), \mathbb{P}(B), \mathbb{P}(A \cap B), \mathbb{P}(A \cup B), \mathbb{P}(A \backslash B)$, and $\mathbb{P}(B \backslash A)$.

2. **Probability laws on countable spaces.** Let Ω be a countable set. Assign a number a_ω to every element $\omega \in \Omega$, so that $a_\omega \geq 0$ for every ω and

$$\sum_{\omega \in \Omega} a_\omega = 1.$$

Then we define a probability law on subsets of Ω as

$$\mathbb{P}(S) = \sum_{\omega \in S} a_\omega$$

Verify that this is indeed a valid probability law. (In fact, any probability law on a countable space has to look like this. If you have extra time, you can try to prove this).

3. **Finite additivity** One of the assumptions in defining a probability law is that $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B)$ for any disjoint A, B.

Deduce from this that

$$\mathbb{P}(A \cup B \cup C) = \mathbb{P}(A) + \mathbb{P}(B) + \mathbb{P}(C)$$

when A, B, C are pairwise disjoint.

If you are familiar with induction, you can prove that

$$\mathbb{P}(A_1 \cup \cdots \cup A_k) = \mathbb{P}(A_1) + \cdots + \mathbb{P}(A_k)$$

for any pairwise disjoint sets $A_1, \ldots, A_k$.