1. **Law of total probability**. Show that, given events A and B with $0 < \mathbb{P}(B) < 1$, we have

$$\mathbb{P}(A) = \mathbb{P}(A|B)\mathbb{P}(B) + \mathbb{P}(A|B^c)\mathbb{P}(B^c)$$

2. **Uses of conditional probability**. In a town of 10,000 people, an outbreak of a rare disease infects 1% of the population (at random). Fortunately, a new medical test has been developed to diagnose the disease. If a person is infected, the test will be positive 99% of the time (this is sometimes called the sensitivity of the test). If a person is healthy the test will be negative 97% of the time (this is sometimes called the specificity of the test).

   (a) A random person is tested, and the test is positive. Without computing anything, what is your best guess for how likely this person is to be infected?

   (b) Write out the meaning of the given conditions in terms of conditional probability. Then compute the probability that this person is infected. (It may be helpful to use the previous problem).

   (c) Compute the probability that a person is infected given that the test is negative.

   (d) Is this test useful?

3. Suppose we have two events, A and B, such that $\mathbb{P}(A) = .8$ and $\mathbb{P}(B) = .9$. Find the maximum and minimum possible values of $\mathbb{P}(A|B)$ and $\mathbb{P}(B|A)$.