Math 184 Week 1

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Recall that for integers n, k, the binomial coefficient $\binom{n}{k}$ counts the number of size-k subsets of a set with n elements. (We take $\binom{n}{k}$ to be zero if $k < 0$ or $k > n$).

These coefficients satisfy some interesting properties. Try to prove some. In particular, it would be nice to find *combinatorial* proofs.

1. $\binom{n}{k} = \binom{n}{n-k}$
2. $\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$
3. $\binom{n}{k-1} < \binom{n}{k}$ whenever $k \leq n/2$
4. $\sum_{k=0}^{n} \binom{n}{k} = 2^n$
5. $\sum_{k=0}^{n} (-1)^k \binom{n}{k} = 0$
6. $\sum_{k=0}^{n} \binom{n}{k}^2 = \binom{2n}{n}$
7. $\sum_{n=0}^{m} \binom{n}{k} = \binom{m+1}{k+1}$