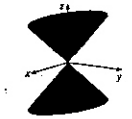
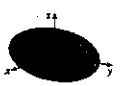
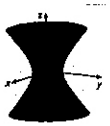
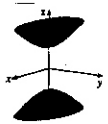
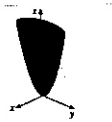
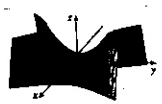
Equations in Three-Dimensions → Cylinders and Quadric Surfaces

If	And	And	Then Surface Is	Example	Surface Shape	Traces of Unique Variable	Traces of Other Variables
Only 2 variables	———	———	Cylinder	$2x^2 - y = 3$			
3 quadratic terms	All positive	No constant	Cone	$ax^2 + by^2 = cz^2$		Point or Circle/Ellipse	Intersecting Lines
3 quadratic terms	All positive	Constant	Ellipsoid/Sphere	$ax^2 + by^2 + cz^2 = d$		Ellipse (use intercepts to find major axis)	Ellipses
3 quadratic terms	1 Negative	———	Hyperboloid of 1 Sheet	$ax^2 + by^2 - cz^2 = d$		Circle/Ellipse	Hyperbolas
3 quadratic terms	2 Negative	———	Hyperboloid of 2 Sheets	$-ax^2 - by^2 + cz^2 = d$		Nothing or Circle/Ellipse	Hyperbolas
2 quadratic terms	All positive	1 Linear term	___ Paraboloid	$ax^2 + by^2 = cz$		Point or Circle/Ellipse	Parabolas in one direction
2 quadratic terms	1 Negative	1 Linear term	Hyperbolic Paraboloid	$ax^2 - by^2 = cz$		Point or Hyperbola	Parabolas in two directions

Note: Constant terms in table are positive real numbers

"Unique" variable determines axis of surface (in the above examples, z is the unique variable) – does not apply to Ellipsoid

If equation involves Quadratic AND Linear terms of one or more variables, the center is not at the origin → Complete the Square