Math 231 – Lecture 1 : Intro & Coördinates

Note Title

All info needed for the course is on Collab. Key things to note:

1. Self-scheduled exams you can retake
2. <u>LOTS</u> of homework. All ungraded.

Office Hours are TBA.

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<u>Today</u>: Review of coordinates and vectors.

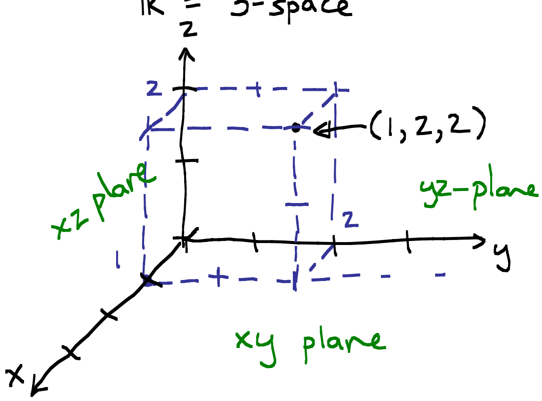
Coordinates: a method of naming every point in the plane or space (etc)

In $\mathbb{R}^2$: (a,b) $\qquad \mathbb{R}^2 =$ xy-plane

In $\mathbb{R}^3$: (a,b,c) $\qquad \mathbb{R}^3 =$ 3-space

Sketching:

Draw a box with one corner at the origin and sides on coord planes



Distance between (x,y,z) and (a,b,c):

$$\sqrt{(x-a)^2+(y-b)^2+(z-c)^2}.$$

Just like in the plane. This is the "Euclidean distance".

<u>Ex</u>: Distance from $(1,1,-1)$ to $(4,5,-13)$:

$$\sqrt{(1-4)^2+(1-5)^2+(-1-{}^-13)^2} = \sqrt{\underbrace{(-3)^2+(-4)^2}_{25}+\underbrace{(12)^2}_{144}} = \sqrt{169} = 13.$$

Knowing distance, get equation for sphere of radius r and centered at (a,b,c)

$$(x-a)^2+(y-b)^2+(z-c)^2=r^2.$$

Sometimes have to rearrange to get this:

<u>Ex</u>: $x^2-4x + y^2-6y + z^2 + 12 = 0$ is the equation of a sphere.

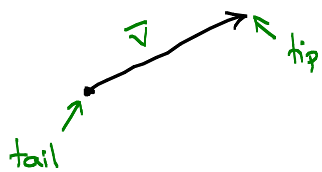
$$(x^2-4x+4) + (y^2-6y+9) + z^2 = -12+4+9$$

complete the square

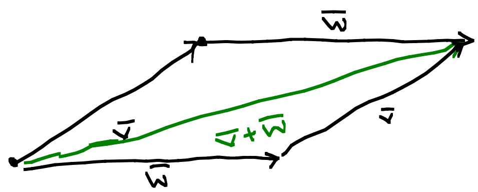
$$(x-2)^2+(y-3)^2+z^2=1. \qquad \text{Center: } (2,3,0)$$
$$\text{radius: } 1$$

A vector has a direction and a length
magnitude

Vectors describe relative positions. They are a kind of rule that produces a new point, the tip, once you specify an old one, the tail:



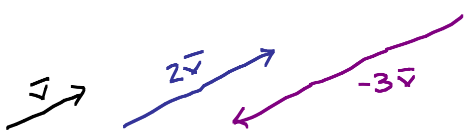
Given two vectors, we can add them:



Place the tail of $\bar{w}$ at the tip of $\bar{v}$ and take the new vector whose tip is that of $\bar{w}$ and whose tail is that of $\bar{v}$.

Given a vector and a real number, can scale the vector.

$c\cdot\bar{v}$ is the vector whose length is $|c|$ times the length of $\bar{v}$ and whose direction is $\begin{cases} \text{that of } \bar{v} & \text{if } c>0 \\ \text{the opposite} & \text{if } c<0 \end{cases}$



It's more convenient to use algebraic quantities than geometric ones. To give "coordinates" to a vector, we place the tail at $(0,0,0)$ and see where the tip lands. If the tip is at (a,b,c), then the coordinates of the vector are $\langle a,b,c\rangle$

note the $\langle\ \rangle$ rather than $(\ ,\ )$. Vectors are not points!

Some things are easy here: addition & scalar mult are coordinate-wise:

$$\langle a,b,c\rangle+\langle d,e,f\rangle=\langle a+d,\, b+e,\, c+f\rangle$$
$$g\cdot\langle a,b,c\rangle=\langle g\cdot a,\, g\cdot b,\, g\cdot c\rangle$$

Also easy to recover the length from the coordinates: $\bar{v} = \langle a, b, c \rangle$

$$|\bar{v}| = d((0,0,0),(a,b,c)) = \sqrt{a^2+b^2+c^2}$$

length pos. of tail pos. of tip

<u>Ex</u>: $\bar{u} = \langle 1,1,1 \rangle$, $\bar{v} = \langle 2,-1,3 \rangle$

$|\bar{u}| = \sqrt{1^2+1^2+1^2} = \sqrt{3}$

$|\bar{v}| = \sqrt{2^2+(-1)^2+3^2} = \sqrt{14}$

$\bar{u}+\bar{v} = \langle 3,0,4 \rangle \Rightarrow |\bar{u}+\bar{v}| = 5$

<u>Ex 2</u>: $\bar{u} = \langle 1,1,1 \rangle$ $a=-1$, $b=2$, $c=3$

$|a\bar{u}| = \sqrt{(-1)^2+(-1)^2+(-1)^2} = \sqrt{3}$

$|b\bar{u}| = \sqrt{2^2+2^2+2^2} = 2\sqrt{3}$

$|c\bar{u}| = \sqrt{3^2+3^2+3^2} = 3\sqrt{3}$

In fact, have

$$|c \cdot \bar{u}| = |c| \cdot |\bar{u}|.$$