Quiz #2

Name: Solutions

February 7, 2008

1. Find the equation of the polynomial of degree 2 whose graph passes through the points $(1,5)$, $(2,7)$, and $(3,9)$.

$$\begin{aligned} 5 &= a_0 + a_1 + a_2 \\ 7 &= a_0 + 2a_1 + 4a_2 \\ 9 &= a_0 + 3a_1 + 9a_2 \end{aligned} \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 1 & 2 & 4 & 7 \\ 1 & 3 & 9 & 9 \end{array}\right] \begin{matrix} \\ -R_1 \\ -R_1 \end{matrix} = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & 1 & 3 & 2 \\ 0 & 2 & 8 & 4 \end{array}\right] \begin{matrix} \\ \\ -2R_2 \end{matrix}$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -2 & 3 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 2 & 0 \end{array}\right] \div 2 \uparrow \uparrow \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{array}\right] \Rightarrow \boxed{y = 3 + 2x}$$

2. Find a basis for the subspace of all vectors orthogonal to $(1,2,-1)$.

$$\vec{x} \cdot (1,2,-1) = 0 \rightsquigarrow x + 2y - z = 0$$

x lead, y, z free : $y = s$, $z = t$

$$\vec{x} = \begin{bmatrix} -2s + t \\ s \\ t \end{bmatrix} = s \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$\text{Basis : } \left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$$

3. Find the norms of and angles between the vectors $(3,-1,2)$ and $(4,1,1)$. You may use inverse trig functions in your answer.

$$\|(3,-1,2)\| = \sqrt{9+1+4} = \sqrt{14}$$

$$\|(4,1,1)\| = \sqrt{16+1+1} = \sqrt{18}$$

$$\cos\theta = \frac{(3,-1,2) \cdot (4,1,1)}{\| \quad \| \cdot \| \quad \|} = \frac{12 - 1 + 2}{\sqrt{14} \cdot \sqrt{18}} = \frac{13}{6\sqrt{7}}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{13}{6\sqrt{7}}\right) \left(= \cos^{-1}\left(\frac{13}{\sqrt{14} \cdot \sqrt{18}}\right)\right)$$