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Assignment 3: Due on Friday, June 10.

• Let $g \in C^1(R)$ be a function, with $g' > 0$. Let $v = g(u)$. If u satisfies

$$\frac{\partial u}{\partial t} = |\nabla u| G(\text{curv}(u)),$$

so does v (contrast invariance or geometric).

• Let $u : R^2 \to R$. The upper level set of u at level $\lambda \in R$ is the set $\chi_\lambda(u) = \{x \in R^2 : \ u(x) \geq \lambda\}$. Show that u can be retrieved by the reconstruction formula

$$u(x) = sup\{\lambda : \ x \in \chi_\lambda(u)\}.$$

• Let $u, v : R^2 \to R$. Assume that u and v have the same level sets, that is for all $\lambda \in R$, there is $\mu \in R$ such that $\chi_\lambda(u) = \chi_\mu(v)$. Let us define g by $g(\lambda) = \sup\Big\{\mu : \ \chi_\lambda(u) = \chi_\mu(v)\Big\}$. Then g is nondecreasing and $v = g \circ u$.

(show first that g is nondecreasing, then show that $v \geq g \circ u$ and that $g \circ u \geq v$).

• Let u be sufficiently smooth and satisfy

$$\frac{\partial u}{\partial t} = |\nabla u| G(\text{curv} u),$$

where $\text{curv} u = \text{div}\Big(\frac{\nabla u}{|\nabla u|}\Big)$ is the curvature operator, and G is a function such that $kG(k) \geq 0$. Show that this flow decreases the total variation of u in time.