Math 270C: Assignment 2

Deadline extension: Due Wednesday, January 26, 2005

[1] Let D_h represent the discrete Laplace operator in two dimensions arising from the discretization of

$$-(u_{xx} + u_{yy}) = f(x, y), \ (x, y) \in \Omega = (0, 1)^2, \ u(x, y) = 0 \text{ if } (x, y) \in \partial\Omega,$$

with the standard five-point, second order, discretization to approximate $u_{xx} + u_{yy}$. Assume that $h = \triangle x = \triangle y$, with $x_0 = 0, x_j = jh, \ j = 0, ..., N$, $x_N = 1$, and $y_0 = 0, y_j = jh, \ j = 0, ..., N, \ y_N = 1$.

(a) Verify (analytically) that the $(N-1)*(N-1)$ eigenvectors of D_h are the discrete functions $s^{(k_1,k_2)}$, whose (i, j)th entry is given by $\sin(k_1\pi x_i)\sin(k_2\pi y_j)$ for a value of k_1 in $\{1, ..., N-1\}$ and k_2 in $\{1, ..., N-1\}$, and $x_i = i(1/N)$, $y_j = j(1/N)$.

(b) Give a formula for the eigenvalues of D_h.

[2] Show that the Gauss-Jacobi iteration can be written in the form $x^{(k+1)} = x^{(k)} + Hr^{(k)}$ where $r^{(k)} = b - Ax^{(k)}$. Repeat for the Gauss-Seidel iteration.

[3] Show that if A is strictly diagonally dominant, then the Gauss-Seidel iteration converges.

[4] Repeat problem [2] from Assignment 1, using now the SOR method with a value of $\omega = 1.5$.