HW#2, 269C, Spring 2006, Vese Due on **Friday, April 28**

[**1**] Find the linear basis functions for the triangle K with vertices at $(0,0)$, $(h,0)$ and $(0,h)$. Show that the corresponding element stiffness matrix is given by

$$\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}$$

Using this result show that the linear system (1.25) of Example 1.1. has the stated form (pages 30-31) there is a typo in the textbook regarding the linear system).

[**2**] Let $\Omega = \{x \in R^2 : \ |x| \leq 1\}$. Show that the function $v(x) = |x|^\alpha$ belongs to $H^1(\Omega)$ if $\alpha > 0$.

[**3**] Consider the problem with an inhomogeneous boundary condition,

$$\begin{cases} -\triangle u = f \text{ in } \Omega, \\ u = u_0 \text{ on } \Gamma = \partial\Omega, \end{cases}$$

where f and u_0 are given. Show that this problem can be given the following equivalent variational formulations:

(V) Find $u \in V(u_0)$ such that $a(u,v) = (f,v)$, $\forall v \in H_0^1(\Omega)$,

(M) Find $u \in V(u_0)$ such that $F(u) \leq F(v)$, $\forall v \in V(u_0)$,

where $V(u_0) = \{v \in H^1(\Omega) : \ v = u_0 \text{ on } \Gamma\}$.

Then formulate a finite element method and prove an error estimate (as in Thm. 1.1, page 24).

Recall: $H_0^1(\Omega) = \{v \in L^2(\Omega), \ \nabla v \in L^2(\Omega)^n, \ v = 0 \text{ on } \partial\Omega\}$, where n is the spatial dimension.

[**4**] Let Ω be a bounded domain in the plane and let the boundary Γ of Ω be devided into two parts Γ_1 and Γ_2. Give a variational formulation of the following problem:

$$\begin{aligned} -\triangle u + u &= f \quad \text{in } \Omega, \\ \frac{\partial u}{\partial \vec{n}} &= g \quad \text{on } \Gamma_1, \\ u &= u_0 \quad \text{on } \Gamma_2, \end{aligned}$$

where f, u_0 and g are given functions. Then formulate a FEM for this problem.