Math 269B, Winter 2003
Instructor: Luminita Vese

Homework #4
Due date: Wednesday, February 12.

REMINDER: Midterm Friday, February 14.

[1] Consider the box scheme

$$\begin{aligned} &\frac{1}{2k}[(v_m^{n+1}+v_{m+1}^{n+1})-(v_m^n+v_{m+1}^n)] \\ +&\frac{a}{2h}[(v_{m+1}^{n+1}-v_m^{n+1})+(v_{m+1}^n-v_m^n)] \\ &=\frac{1}{4}(f_{m+1}^{n+1}+f_m^{n+1}+f_{m+1}^n+f_m^n) \end{aligned}$$

(a) Show that the scheme is an approximation to the one-way wave equation $u_t + au_x = f$ that is accurate of order (2,2)

(b) Show that the scheme is stable for all values of λ.

[2] Consider the system

$$\begin{cases} u_t = av_x \\ v_t = au_x. \end{cases}$$

(a) Give a single PDE equivalent with this system.

(b) Apply the Lax-Friedrichs scheme to the system. Is the obtained system of finite differences consistent with the system of PDE's ?

(c) Apply von Neumann stability analysis to the vector scheme, in the case when $\lambda = \frac{k}{h} = constant$, substituting

$$\begin{pmatrix} u_m^n \\ v_m^n \end{pmatrix} = g^n e^{im\theta} \begin{pmatrix} u^0 \\ v^0 \end{pmatrix}$$

in the difference vector equation, where $\begin{pmatrix} u^0 \\ v^0 \end{pmatrix}$ is a constant vector both in space and in time.

[3] Consider the nonlinear equation $u_t + u_x = \cos^2 u$, approximated by the Lax-Wendroff scheme with $R_{k,h} f_m^n = f_m^n$, treating the $\cos^2 u$ term as $f(t,x)$. Show that the obtained scheme is first order accurate (use $\lambda = \frac{k}{h}$).