Math 164, Vese: Homework #6, due on Friday, February 20

• We have a midterm exam on Friday, February 20, 2.00pm-2.50pm. Sample midterm problems with solutions are posted on the class webpage. This will be a closed notes and closed book exam. No calculators are allowed.

• Sections covered for the midterm: 1, 2.2-2.3 (except 2.3.1), 3.1, 3.2, 4.1-4.4, 5.2, 6.1-6.2.

[1] Find the dual of
maximize $z = 6x_1 - 3x_2 - 2x_3 + 5x_4$,
subject to
$4x_1 + 3x_2 - 8x_3 + 7x_4 = 11$
$3x_1 + 2x_2 + 7x_3 + 6x_4 \geq 23$
$7x_1 + 4x_2 + 3x_3 + 2x_4 \leq 12$
$x_1, x_2 \geq 0$, $x_3 \leq 0$, x_4 free
Verify that the dual of the dual is the primal.

[2] Find the dual to the problem
minimize $z = c^T x$,
subject to $b_1 \leq Ax \leq b_2$,
$x \geq 0$.

[3] Consider the linear program

$$\text{maximize } z = -x_1 - x_2, \text{ subject to } \begin{cases} -x_1 + x_2 \geq 1 \\ 2x_1 - x_2 \leq 2 \\ x_1, x_2 \geq 0. \end{cases}$$

Find the dual to the problem. Solve the primal and the dual graphically, and verify that the results of the strong duality theorem hold.

[4] Consider the linear program

$$\begin{array}{ll} \text{minimize} & z = 2x_1 + 9x_2 + 3x_3 \\ \text{subject to} & -2x_1 + 2x_2 + x_3 \geq 1 \\ & x_1 + 4x_2 - x_3 \geq 1 \\ & x_1,\ x_2,\ x_3 \geq 0. \end{array}$$

(a) Find the dual to this problem and solve the dual problem graphically.
(b) Use complementary slackness to obtain the solution to the primal.

[5] Consider the primal linear programming problem
Minimize $z = c^T x$
subject to $Ax \leq b$,
$x \geq 0$.

Assume that this problem and its dual are both feasible. Let x_* be an optimal solution vector to the primal, let z_* be its associated objective value, and let y_* be an optimal solution vector to the dual problem. Show that $z_* = y_*^T A x_*$.