Math 95 – Transition to Upper-division Math
Course notes

Tyler Arant

Disclaimer. These notes have been used twice previously for Math 95 course at UCLA. The students in those courses did a great job finding typos, and their efforts were very much appreciated. But there's no doubt errors and typos still exist, and they are entirely the author's fault. Please keep this in mind, and do not hesitate to point any out if you find them.

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1 Introduction

Lower-division mathematics courses are mostly about mastering computational algorithms, e.g.,

- how to find the local extrema of a differentiable function;
- how to find the eigenvalues and eigenvectors of a square matrix;
- how to solve a second-order, linear, constant coefficient, homogenous differential equation, etc.

Although computation and algorithms are an important part of mathematics, upper-division mathematics courses require more from students. As most students are aware, upper-division courses require the writing of proofs. However, there are other types of skills that are involved, for example:

- A much bigger emphasis on mathematical definitions. These definitions must be memorized and understood backwards and forwards. This point really cannot be understated. Your first (and perhaps most important) obligation in a mathematics course is to learn all the definitions presented in the course. Often, exams will ask for students to write down definitions of important concepts. In my experience grading, the students who fail to write down correct definitions on the exam are almost guaranteed to not do well on the exam.

- Understanding the "why" and not just the "how". Moving past just memorizing an algorithm, upper-division math questions sometimes require you to reconfigure knowledge in novel ways (which are not explicitly presented in lecture or the textbook). But this reconfiguration of knowledge requires an understanding of why the other facts/algorithms/computations/theorems are true, so that you can produce some (usually minor) variation that works in a new situation. Related to this bullet point is the following:

- It is not just about boxed formulas! A very common method of studying for lower-division math exams is to go through the textbook and memorize all the boxed formula that you find. This method is unsuccessful in upper-division course for a few reasons; (1) for the reasons mentioned in the previous bullet point regarding understanding why the formula is true, and (2) this method tends to overlook learning the context of when the formula is applicable. Without a proper understanding of the "hypothesis" required to make the theorem true, you are very likely to incorrectly apply the theorem/formula. Consider, for example, Clairaut's theorem. Most students probably remember the formula

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x},$$

  but don't realize that the this equality can, in fact, fail! Clairaut's theorem only ensures the equality holds under the condition that the functions $\frac{\partial^2 f}{\partial x \partial y}, \frac{\partial^2 f}{\partial y \partial x}$ are continuous. This is a classic example where the "boxed formula" does not tell the whole story.

This course will focus on learning all of these skills and more. We begin with studying sets and logic, which are both fundamental to any area of modern mathematics.