Homework 4. Problem 1

eq. (5.30):

$$y_{i+1} = y_i + h\left[f_i + \frac{1}{2}\nabla f_i + \frac{5}{12}\nabla^2 f_i + \frac{3}{8}\nabla^3 f_i\right.$$

$$= y_i + h\left[f_i + \frac{1}{2}(f_i - f_{i-1}) + \frac{5}{12}(f_i - 2f_{i-1} + f_{i-2}) + \frac{3}{8}(f_i - 3f_{i-1} + 3f_{i-2} - f_i\right.$$

$$= y_i + \frac{h}{24}\left[55 f_i - 59 f_{i-1} + 37 f_{i-2} - 9 f_{i-3}\right]$$

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Homework 4. Problem 2

$$\frac{dy}{dt} = f = -20y + 20t^2 + 2t$$

define: $F(w) = w - w_j - \frac{h}{2}\left[f(t_{j+1}, w) + f(t_j, w_j)\right]$

Apply Newton's Method (c.f. notes):

$$w_{j+1}^{(k)} = w_{j+1}^{(k-1)} - \frac{w_{j+1}^{(k-1)} - w_j - \frac{h}{2}\left[f(t_{j+1}, w_{j+1}^{(k-1)}) + f(t_j, w_j)\right]}{1 - \frac{h}{2} f_y(t_{j+1}, w_{j+1}^{(k-1)})}$$

Now plug in f, and $f_y = -20$

$$w_{j+1}^{(k)} = w_{j+1}^{(k-1)} - \frac{w_{j+1}^{(k-1)} - w_j - \frac{h}{2}\left[-20w_j + 20t_j^2 + 2t_j - 20w_{j+1}^{(k-1)} + 20t_{j+1}^2 - 2t_{j+1}\right]}{1 + 10h}$$

$$= w_{j+1}^{(k-1)} - \frac{w_{j+1}^{(k-1)}(1+10h)}{(1+10h)} + \frac{1}{(1+10h)}\left(w_j - 10hw_j + 20t_j^2 + 2t_j + 20t_{j+1}^2 + 2t_{j+1}\right)$$

$$= w_j \frac{1-10h}{1+10h} + \frac{h}{1+10h}\left(10t_j^2 + 10t_{j+1}^2 + t_j + t_{j+1}\right)$$