Homework #1

Example:

5.1.1.a) $y' = y\cos t$, $0 \le t \le 1$; $y(0) = 1$

1.a)

$f(t,y) = y \cdot \cos(t)$

$|f(t,y_1) - f(t,y_2)| = |y_1\cos t - y_2\cos t| = |y_1 - y_2|\cos t$

$\le 1 \cdot |y_1 - y_2|$

1.b) b) $y' = \frac{2}{t}y + t^2 e^t$ $1 \le t \le 2$; $y(1) = 0$

$|f(t,y_1) - f(t,y_2)| = |\frac{2}{t}y_1 + t^2e^t - \frac{2}{t}y_2 - t^2e^t|$

$= |\frac{2}{t}(y_1 - y_2)| = \frac{2}{t}|y_1 - y_2| \le 2|y_1 - y_2|$

1.c) d) $y' = \frac{4t^3y}{1+t^4}$ $0 \le t \le 1$ $y(0) = 1$

$|f(t,y_1) - f(t,y_2)| = |\frac{4t^3y_1}{1+t^4} - \frac{4t^3y_2}{1+t^4}|$

$= |\frac{4t^3}{1+t^4}| \, |y_1 - y_2| \le 2|y_1 - y_2|$.

1.b)

$y' = 9y$ $0 \le t \le t_{max}$

$|f(t,y_1) - f(t,y_2)| = |9y_1 - 9y_2| = 9|y_1 - y_2|$.

Lipschitz const.

hint: follow proof of 5.9.1 what is δ_i, t, a_i now?

Proof Theorem 5.10.

This is analoguous to Theorem 5.9.

$$y_{i+1} - w_{i+1} = y_i - w_i - \delta_{i+1} + h\left[f(t_i, y_i) - f(t_i, w_i)\right] + \frac{h^2}{2} y''$$

$$\leq (y_i - w_i) + h\left[f(t_i, y_i) - f(t_i, w_i)\right] + \frac{h^2}{2} y''(\xi) + \delta$$

since all $|\delta_i| \leq \delta$

all terms are the same as for Theorem, Except δ.

$\rightarrow$

$$\underbrace{|y_{i+1} - w_{i+1}|}_{a_{i+1}} \leq \underbrace{(1 + hL)}_{1+s} \underbrace{|y_i - w_i|}_{a_i} + \underbrace{\frac{h^2 M}{t} + \delta}_{t}$$

5.8.

$\rightarrow$

$$|y_{i+1} - w_{i+1}| \leq e^{(i+1)Lh}\left[\underbrace{|y_0 - w_0|}_{|\delta_0|} + \left(\frac{h^2 M}{2hL} + \frac{\delta}{hL}\right)\right] - \frac{h^2 M}{2hL} - \frac{\delta}{hL}$$

$$\leq \left(e^{(i+1)hL} - 1\right)\frac{1}{L}\left(\frac{hM}{2} + \frac{\delta}{h}\right) + |\delta_0| e^{(i+1)hL}$$

$\rightarrow$

$$|y_i - w_i| \leq \frac{1}{L}\left(\frac{hM}{2} + \frac{\delta}{h}\right)\left[e^{L(t_i - a)} - 1\right] + |\delta_0| e^{L(t_i - a)}$$

q.e.d.