2016 Distinguished Lecture Series
UCLA Department of Mathematics

Benedict Gross
Harvard University

Lecture 1: **The rank of elliptic curves**

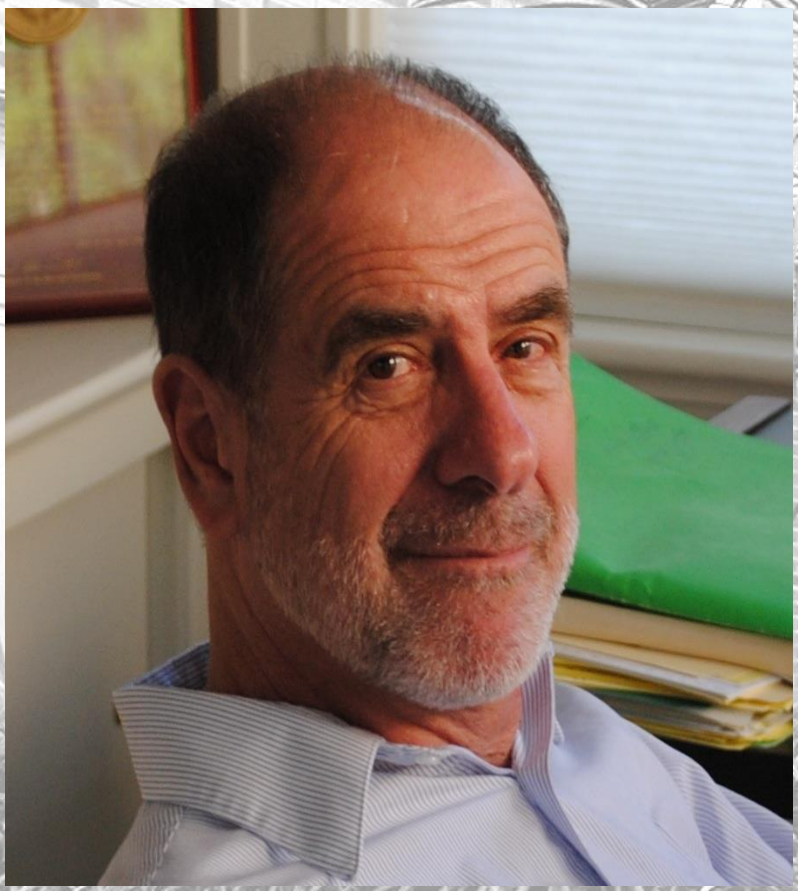
Abstract: Elliptic curves, or cubic equations in two variables, have been a central object of study in number theory since the time of Fermat. The set of their rational points forms an abelian group, which Mordell proved was finitely generated. Many of the interesting open questions in the field concern the rank of this group. In this talk, I will review the conjecture of Birch and Swinnerton-Dyer, and summarize the progress that has been made in the direction of a proof. I will also discuss approaches to study the average rank, for all elliptic curves over Q.

Lecture 2: **The arithmetic of hyperelliptic curves**

Abstract: Hyperelliptic curves first appeared in the work of Abel on integration, where he defined their genus g. Every such curve of genus g has an affine equation of the form $y^2 = F(x)$, where $F(x)$ is a separable polynomial of degree $2g+2$. Abel and his contemporaries studied these curves over the real and complex numbers; in this talk I will focus on the case when the curve is defined over the rational numbers (or equivalently, when the polynomial $F(x)$ has rational coefficients). In that case, an important invariant is the set of rational solutions. When the genus is at least 2, Faltings proved that this set is finite. In fact, one can now show that this set is usually empty, and in many cases, there are no solutions over any odd degree extension of Q.

Lecture 3: **Pencils of quadrics and the Jacobians of hyperelliptic curves**

Abstract: Beyond linear subspaces in projective space, the next simplest subvarieties are quadrics, which are the hypersurfaces of degree 2. These are easily classified by a discrete invariant (the rank) over the complex numbers, but once one takes a pair of quadrics, given by two symmetric matrices A and B, the situation becomes more interesting. The most general pencils $Ax - By$ are those where the binary form $\det(Ax - By)$ has a non-zero discriminant. I will discuss how this theory is related to certain coverings of the Jacobians of hyperelliptic curves, and how pencils can be used to study the arithmetic of the curve.



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Lecture 1
Tuesday, April 26, 2016
3:00 - 3:50 pm
MS 6627

Lecture 2
Wednesday, April 27, 2016
3:00 - 3:50 pm
MS 6627

Lecture 3
Thursday, April 28, 2016
4:15 – 5:00 pm
MS 6627

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