2013-2014 Distinguished Lecture Series
UCLA Department of Mathematics

Differential Systems and Holonomy

Lecture 1: Rolling constraints and geometry

Abstract: The problem of navigating in a space with rolling constraints arises in many contexts, from parking a car to robot hands manipulating small objects. I will consider some examples and their analysis from the point of view of geometry, including an application of Cartan's famous 5-variables paper of 1910, which applies to the problem of one surface rolling over another without twisting or slipping and in which the first exceptional Lie group makes a surprising appearance.

Lecture 2: Prescribing holonomy groups via differential equations

Abstract: While the appearance of the holonomy group in Riemannian geometry is well-known, the history contains some surprises, including Cartan's first nontrivial example in 1926, which is now recognized to be that of a Ricci-flat Kähler metric in complex dimension~2. Many problems in geometry can be rephrased as the problem of locally prescribing a given group as holonomy, and this can be reduced to a PDE problem in a number of ways, but most of these lead to PDE that are either degenerate or overdetermined in some way, so the methods of exterior differential systems turn out to be essentially involved. I will show how these problems can be expressed as existence problems for systems of Cartan structure equations and then analyzed using Cartan's generalization of Lie's third fundamental theorem.

Lecture 3: Applications to the geometry of moduli spaces

Abstract: One of the aspects of Penrose's twistor theory is expressing the solutions to certain systems of nonlinear PDE as geometries on moduli spaces of rational curves subject to some differential constraints, for example, Penrose's solution of the self-dual Einstein equations in terms of rational curves with a specified normal bundle in a three dimensional complex space. In this final lecture, I will discuss a unified picture of a number of these classical differential geometry problems in which they can be expressed as prescribed holonomy problems and explain what the techniques of exterior differential systems has been able to say so far about them.

Lecture 1
Tuesday, May 20, 2014
2:00 - 2:50 pm
MS 6627

Lecture 2
Wednesday, May 21, 2014
2:00 - 2:50 pm
MS 6627

Lecture 3
Thursday, May 22, 2014
2:00 – 2:50 pm
MS 6627

Robert Bryant
Duke University

UCLA